An Eternal History of Frozen Time?

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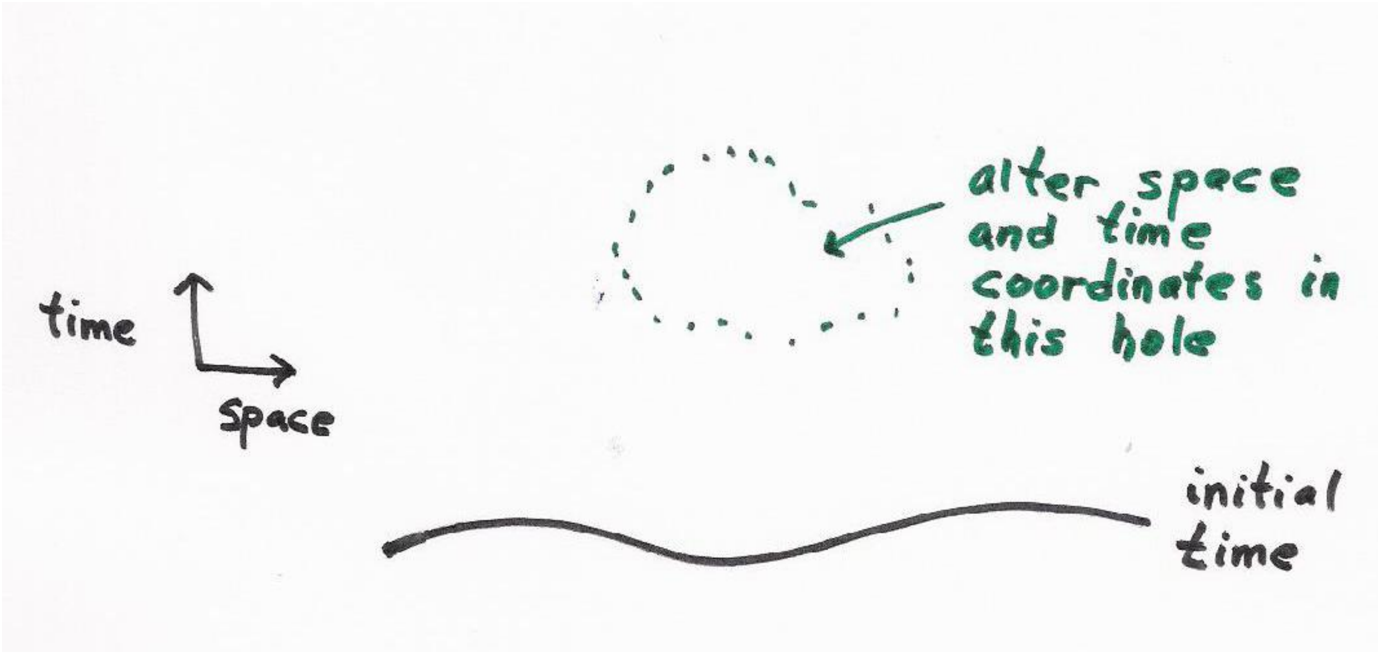
Mini-symposium on the Nature of Time
Austin College

Plan of Talk

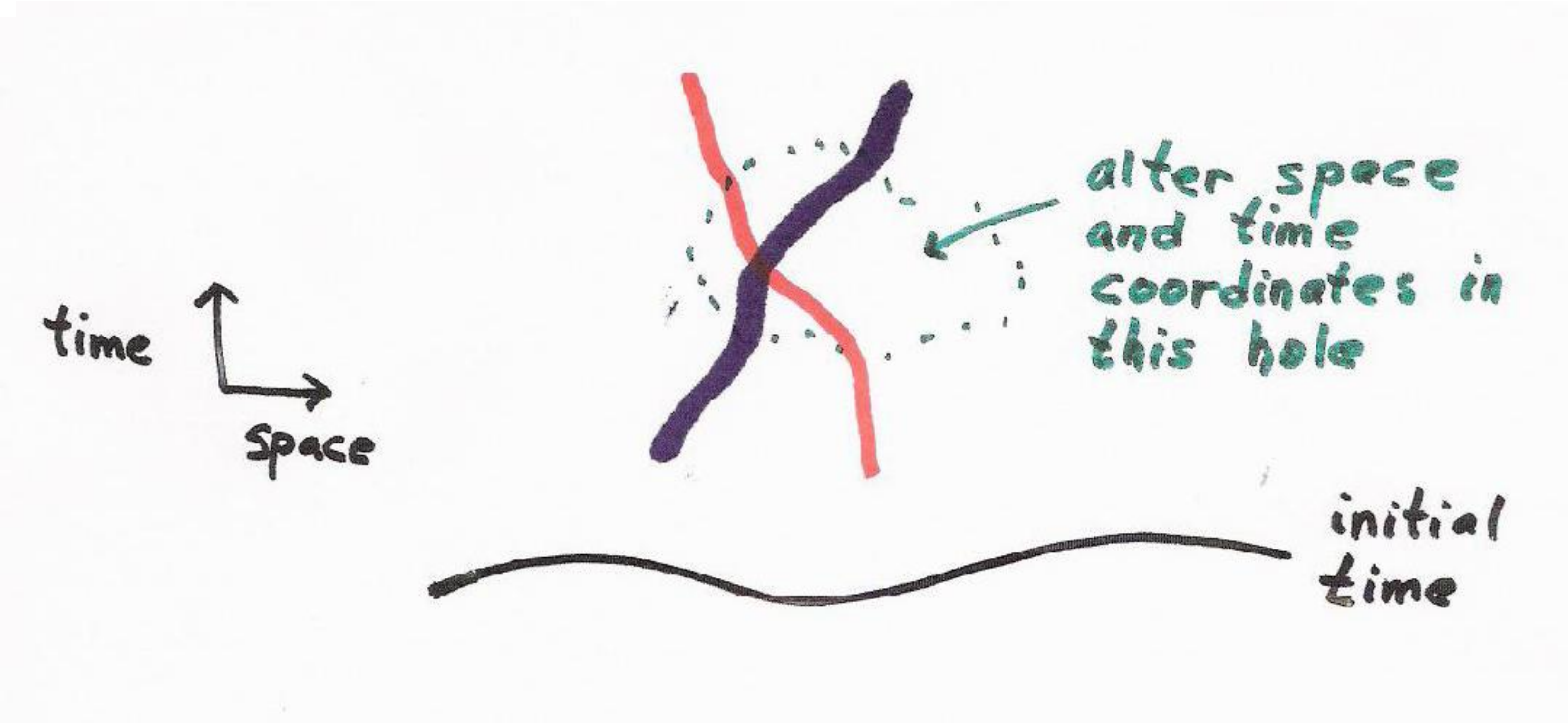
1. Coordinate symmetry and Einstein's hole argument
2. Early history of initial value formulation of general relativity
3. Peter Bergmann and Paul Dirac
4. Frozen time
5. Symmetry analysis by Bergmann and Art Komar
6. Symmetry analysis of Pons, Shepley and Salisbury
7. Relative ontological time in Einstein's universe?
8. Implications for quantum gravity

I. Coordinate symmetry and Einstein's hole argument



Einstein (1913): change of space and time coordinates in the hole produces changes in "physical" quantities in the hole, but no changes at the initial time. But since the dynamical equations have the same form under arbitrary changes in coordinates (general covariance), the transformed quantities represent new solutions - but with the same starting assumptions. Looks like breakdown of determinism.

Einstein finally in 1915 embraced coordinate symmetry (general covariance) by proclaiming that only material coincidences were real. Particle collisions identify points (events) in spacetime



I. Early history of initial value formulation of general relativity

Obstacles on the road to quantum electrodynamics - symmetry results in a vanishing momentum

• Theories of Werner Heisenberg and Wolfgang Pauli (1929-30)

• Pauli suggestion to Leon Rosenfeld leads to Rosenfeld’s groundbreaking paper of 1930, and first attempt to construct a quantum field theory of electricity, magnetism, and gravity.

Zur Quantelung der Wellenfelder

Von L. Rosenfeld

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1. Die erste Heisenberg-Paulische Methode ist ein rein analytischer Kunstgriff.[2]) Man fügt zur Lagrangefunktion gewisse Zusatzglieder hinzu, die mit einem kleinen Parameter ε multipliziert sind und bewirken, daß der obenerwähnte Impuls nicht mehr verschwindet. In den Schlußresultaten muß man dann zum Limes $\varepsilon = 0$ übergehen. Die ε-Glieder führen aber zu unphysikalischen Rechenkomplikationen[3]) und zerstören die charakteristische Invarianz der Lagrangefunktion gegenüber der Eichinvarianzgruppe.

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Annalen der Physik. 5. Folge. 5. 8

Heisenberg and Rosenfeld

Rosenfeld's formalism

- Rosenfeld's "constrained dynamics" formalism showed how arbitrary function appear in time evolution from an initial moment
- He showed how the dynamics of general relativity leads to further restrictions on possible values of initial gravitational and material variables
- He was the first to establish a relation between changes in spacetime coordinates and changes in gravitational and material variables at a fixed time
- Rosenfeld's pioneering work was largely ignored, and many of his results were independently rediscovered about two decades by Peter Bergmann and Paul Dirac

III. Peter Bergmann and Paul Dirac

Brief Bergmann biography

- Born Berlin-Charlottenburg 1915
- Mother Dr. Emmy Bergmann moved with children to Freiburg 1922 - she and sister emigrated to Israel 1935
- Father Dr. Max Bergmann 1921 - 1933 head of Institut für Lederforschung, Dresden (now Max Bergmann Zentrum für Biomaterialen)
- Prague, Charles University degree 1936
- Einstein Assistant 1936 - 1941: unified field theory
- Syracuse University 1947 - 1982
- Died October 2002

- Bergmann began effort at Syracuse University in 1949 to create a quantum theory of gravity

Excerpt from letter of recommendation for Bergmann from Albert Einstein in 1954:

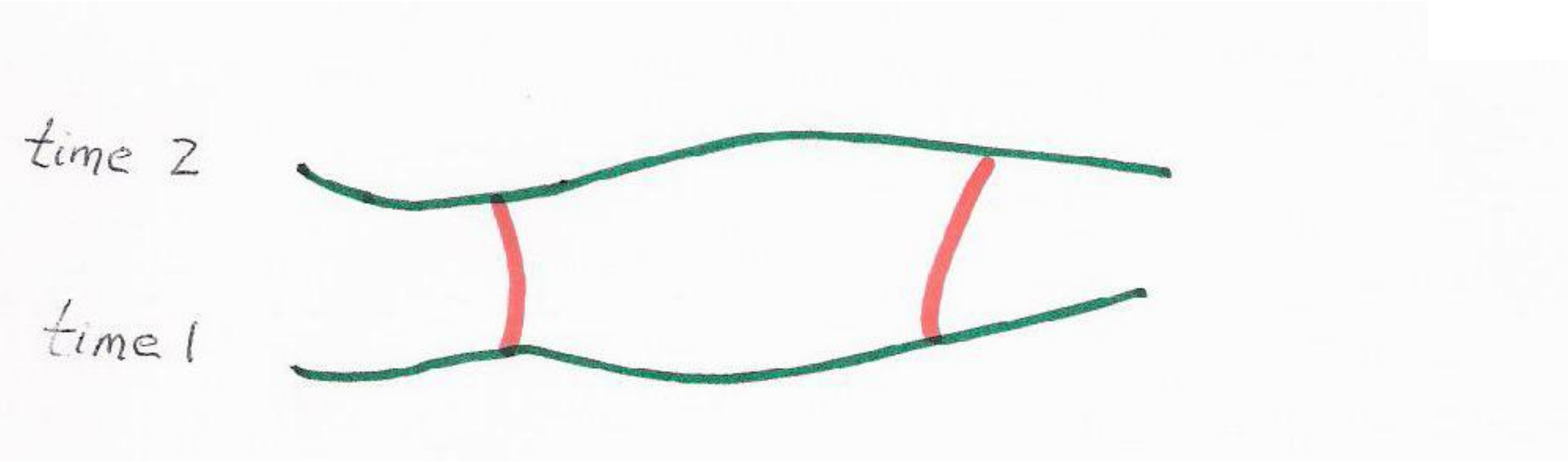
> All physicists are convinced of the high truth value of the probabilistic quantum theory and of the general theory of relativity. These two theories are however based on independent conceptual foundations, and their combination to a unified logical system has so far resisted all attempts in this direction. There are presently only few theoretical physicists who have penetrated deeply enough into both theories to be able to undertake such an attempt at all. Dr. Bergmann is one of the few who are completely at home with both theories.

- Initial task was to reformulate relativity in terms of fields and momenta. They would then be promoted following the canonical procedure to quantum mechanical measurement operations

- Focus from the beginning was on the transformation of fields and momenta that resulted from general coordinate transformations

Paul Dirac's breakthrough

- Dirac, "The theory of gravitation in Hamiltonian form", Proc. Roy. Soc. **A246**, 327 (1958)
  - Form of constraints and dynamical laws is simplified
  - Lapse and shift are abandoned as canonical variables



IV. Frozen time

- First suggestion by Bergmann
- Bergmann - Dirac correspondence

Dear Professor Dirac:

I have just studied your paper that appeared in the May 1 issue of the Physical Review. I am writing you, first to ask you for a reprint when they are available, but I should also like to make a few comments.

(1) The objections that Professor Lichnerowicz and I raised at the end of your lecture at Royaumont, whether or not they were valid then, certainly do not apply to the work that you have published here. Regardless of the motive of introducing the metric g_{rs} on the initial hypersurface, the canonical transformation that you first published a year ago to simplify and kill the primary constraints, is both legimate and sucessful. At this stage the total number of canonical field variables is reduced from twenty to twelve.

Excerpt of letter from Bergmann to Dirac dated October 9, 1959

(3) When I discussed your paper at a Stevens conference yesterday, two more questions arose, which I should like to submit to you: To me it appeared that because you use the Hamiltonian constraint H_L to eliminate one of the non-substantive field variables, κ , in the final formulation of the theory your Hamiltonian vanishes strongly, and hence all the final field variables, i.e. $\tilde{e}^{rs}, \tilde{p}^{rs}$, are "frozen" (constants of the motion). I should not consider that as a source of embarrassment, but Jim Anderson says that in talking to you he found that you now look at the situation a bit differently. Could you enlighten me? If you have no objection, I should communicate your reply to Anderson and a few other participants in the discussion.

If ~~you~~ the conditions you introduce to fix the surface are such that only one surface satisfies the conditions, then the surface cannot move at all, the Hamiltonian will vanish strongly and all dynamical variables will be frozen. However, one may introduce conditions which allow an infinity of roughly parallel surfaces. The surface can then move with one degree of freedom and there ~~will~~ must be one non-vanishing Hamiltonian that generates this motion.

I believe my condition $g_{rs}p^{rs} \approx 0$ is of this second type, or maybe it allows also a more general motion of the surface corresponding roughly to Lorentz transformations. The non-vanishing Hamiltonian one would get by subtracting a divergence term from the density of the Hamiltonian.

Excerpt of response from Dirac to Bergmann, dated November 11, 1959

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Excerpt of letter from Peter Bergmann to Nathan Rosen, dated September 26, 1973

Dirac is perhaps the last of the really great pioneers that created today's physics. Though he may not be able to last through a heavy conference schedule, he will take in a few papers a day every day for a week, and he will make very helpful and acute comments on occasion. His presence will, of course, lend prestige to GRG7, but he will be a real asset as a physicist. Having through an extended period wrestled with the same problems that he succeeded in solving - a viable Hamiltonian version of general relativity, I have the profoundest respect for his genius, second only (in my personal experience) to Einstein. I think that you should act soon.

V. Analysis by Bergmann and Art Komar- 1972

- First analysis of effects of successive coordinate transformations
- Discovery that cumulative transformations must depend on the gravitational field - first hint of relational time
- Suggestion of intrinsic time - Rovelli essentially reprises this idea in the 1990's

VI. Analysis of Pons, Shepley and Salisbury: 1997 -

- Evolution in time is not a symmetry with regard to the evolution of fields and momenta - so observables need not be constants of the motion

- The full symmetry set of coordinate symmetry transformations is implementable, but the lapse must be retained as a variable

- The Rovelli program of relational time can be realized in general relativity through the use of intrinsic time. There is no paradoxical variation in time without time.

VII. Relative ontological time?

- Can time be real and relational? time. There is no paradoxical variation in time without time.

V - Leon Rosenfeld and his pioneering work

- Born Belgium 1904
- Doctorate Liege 1926
- Research in Paris, Göttingen, Zurich 1926-1930
- Taught theoretical physics at Liege, Utrecht, Manchester, Copenhagen 1940-1974
- Collaborators and correspondents: Bohr, Pauli, de Broglie, Dirac, Heisenberg, Infeld, Klein …
- Died October 1974

Rosenfeld (right, standing)
at 1933 Solvay Meeting

The initial value (Hamiltonian) formulation of electromagnetism

- Required to take canonical route to quantization of the elctromagnetic field
- Must rewrite equations of motion so as to contain only first derivatives with respect to time. This is done by defining new variables, the "momenta", in terms of the velocities.

  For example, in the case of the object in free fall, let $p = \frac{dz}{dt}$

  then this definition plus the equation of motion $\frac{dp}{dt} = -g$

  become the Hamiltonian equations of motion. These equations of motion are determined by a "Hamiltonian"

$$H = gz + \frac{1}{2} p^2$$

- But there is a problem with electromagnetism. One of the canonical momenta (the one associated with the time derivative of the electrostatic potential) vanishes identically!

Proposals for dealing with vanishing momentum:

Heisenberg/Pauli formalism (1929-1930)

• Add non-gauge invariant term to Lagrangian (destroys local symmetry)

• Or set *V = constant* (destroys manifest Lorentz invariance)

Pauli: "Ich warne Neugierige"

Rosenfeld's debt to Pauli: "*As I was investigating these relations in the especially instructive example of gravitation theory, Professor Pauli helpfully indicated to me the principles of a simpler and more natural manner of applying the Hamiltionian procedure in the presence of identities*" (My translation from Rosenfeld's 1930 paper)

Rosenfeld's formal constraint analysis in "On the quantization of wave fields", Annalen der Physik 1930

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- Local symmetries always lead to
  - non-unique evolution in time
  - constraining relations among variables and associated momenta
  - Hamiltonian (from which equations of motion are determined) constructed using the constraints
  - vanishing of Hamiltonian if, as in general relativity, the equations of motion take the same form for arbitrary choices of the time coordinate
- Rosenfeld was first to consider how to implement local symmetry-induced transformations on Hamiltonian variables
- Rosenfeld's dynamical model - gravitation with a charged spinorial

Dirac field source

Origins of the model

- Weyl/Fock coupling of Dirac field with gravity - 1929
- Tetrads and Weyl's reinterpretation of gauge symmetry
  See analyses by Scholz (physics/0409158) and Straumann (hep-ph/0509116)

VI -The symmetry under general coordinate transformations of general relativity

- There is no preferred way of assigning spatial or temporal coordinates in general relativity
- But - we do now have a way - following the pioneering work of Rosenfeld, Bergmann, and Dirac - of tracking the evolution from an initial instant for any choice we wish to make for a temporal coordinate
- Bergmann and Komar (1972), following up on the work of Paul Dirac (1958), made the first step in understanding how general coordinate symmetry is preserved in the initial value (Hamiltonian) version of general relativity
- Pons, Salisbury and Shepley (1997-2001) showed that the underlying initial value (Hamiltonian) symmetry is relational in the sense that the symmetries depend not only on arbitrary spacetime functions - <u>but necessarily also on the physical gravitational field</u>.
- Pons and Salisbury (2005) explained how to construct a univocal relational time, exploiting the newly discovered Hamiltonian symmetry. An "intrinsic" time is defined using an appropriate function of physical fields.

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VI - A cosmological example of relative ontological(?) time

- Isotropic expanding universe containing a massless scalar field, with two gravitational (metric) variables, the spatial metric (expansion factor) a and the lapse function N
- The Hamiltonian model is symmetric under the small time transformation

$$t' = t - \frac{\xi(t)}{N(t)}$$

- Choose the square of the expansion factor as the intrinsic time since it increases monotonically with coordinate time
- The model fixes a unique correlation between the value of a^2 and the value of the scalar field
- It can be shown explicitly that the resulting fields are invariant under the group of transformations given above - thus we have true evolution in intrinsic time, but only when there is stuff in the universe!

VII - Implications for quantum gravity

- In the loop approach to quantum gravity a^2 can take only certain discrete values, determined in terms of the Planck time (about 10^{-43} seconds)
- Although most researchers in the field are satisfied that no notion of temporal evolution need be present in the Planckian era, we maintain that one can sensibly construct a generalized Schroedinger quantum time stepping.

- Most of the quantum relativity community is still convinced that quantum time is “frozen”, yet most also recognize the possibility of non-trivial evolution in “parameter” time.
- But our intrinsic evolution is <u>real</u>, and the evolving variables are observables in the sense that they do not change under arbitrary (permissable) transformations in the time coordinate

Generalized time-dependent Schrödinger equation (gr-qc/0702132)

Use discrete time eigenvalues from loop gravity

$$t_k = \frac{k}{6}, \quad k = 0,1,2, \mathrm{K}$$

$$\text{Let } \left|\psi(\phi, t_{k+1})\right\rangle = \left(1 - \frac{i\Delta t H}{\mathrm{h}}\right)\left|\psi(\phi, t_k)\right\rangle = \left(1 - \frac{9i}{2\mathrm{h}^2(k+1)}\right)\left|\psi(\phi, t_k)\right\rangle$$