The Origins and Impact of the Principle of Least Action of de Maupertuis

Donald Salisbury,[1,2*] Jürgen Renn[1]

[1]Max Planck Institute for the History of Science,
Boltzmannstrasse 22, 14195 Berlin, Germany
[2]Austin College, 900 North Grand Ave, Sherman, Texas 75090, USA

*To whom correspondence should be addressed; E-mail: dsalisbury@austincollege.edu.

Pierre Louis de Maupertuis was arguably the first modern scientist to seek and find a mathematical principle that could unify optical and mechanical phenomena. Variations and reinterpretations of his principle of least action indeed lie at the core of contemporary efforts to marry quantum mechanics and relativity theory. In fact, the implications and controversies associated with these attempts are no less striking than what we witness in the Enlightenment world of de Maupertuis.

I want to examine today a case study in the dynamics of scientific development. The idea I have chosen is especially appealing since the situation today regarding its interpretation and usage is as fraught with controversy, as firmly rooted in contemporary knowledge structures, and as suggestive of hints into imminent paradigm shifts as when it was introduced by Maupertuis in the mid-eighteenth century. The principle of least action was introduced by Maupertuis in 1744, and there is little doubt today that when properly interpreted it will provide the key to an eventual unification of quantum mechanics and Einsteins theory of gravity. This is an ideal study to undertake with the students since it deals with a basic contemporary unresolved puzzle. They therefore have less reason to dismiss outright the obviously "errant" science and are more open to a critical reading of an "ancient" text. This in turn affords them the possibility of sensing the intellectual contextual origins of the text but also beginning to appreciate the role that context plays in the current debate around quantum gravity. Indeed, what ultimately results is an understanding of scientific discovery as firmly rooted in the liberal arts, and this serves not only as a lesson to students but also to scientific professionals. We professionals are in a very real sense doing history in our respective scientific disciplines, and I would argue that the degree to which we are cognizant of this fact will have a bearing on our success.

Let me say right up front that the fundamental problem that we face today in attempting to marry quantum mechanics with Einsteins general theory of relativity is that that two theories are built on contradictory conceptual structures. Einsteins theory posits an egalitarian role for space and time, while in quantum mechanics those bizarre quantum leaps manifest themselves in a fundamental way in the spatial position of matter and not in the time of observation. What seems to be required is a theory in which space and time enjoy an equal status yet exhibit the uncertainties that are characteristic of quantum mechanics.

Pierre Louis de Maupertuis was born in the French port of Saint-Malo in 1698 and died in 1759 in self-imposed exile in Basel toward the end of the Seven Year War. He was torn between allegiance to two patrons, King Louis IV of France and Frederick the Great of Prussia. In France he had been elected a member of both the Paris Academy of Science and the Académie Française and had been lured to Berlin in 1746 by Frederick II to become the first president of the Berlin Academy of Sciences and Belles-Lettres. His father René Moreau had been ennobled in 1708 by Louis IV in recognition his fathers privateering attacks of English shipping, and it is likely the familys commercial parvenu status that contributed to Maupertuis lifelong successful effort to elevate his name in the socially connected scientific and literary circles of Paris. In addition to his technical treatises he published several tracts of a semi-popular nature for a predominantly female literary salon audience. He made a name for himself in the pre-Berlin period following his successful expedition to Lapland in 1736 in which, using advanced English sighting instruments, he was able to prove, as predicted by Newtonian theory, that the earth is slightly shaped into the form of an ellipsoid that is flattened near the poles.

Though he became a defender of Newton on the continent, opposing mostly behind the scenes and often through anonymous publications the Descartian view of continuum mechanics, he did express discomfort with the Newtonian notion of force acting at a distance. He was, however, firmly attached to the notion of particles propagating in a void. Amongst those he included particles of light, and he was concerned not only that there did not appear to exist a unifying principle that could predict the outcome of material particle collisions, but also that the then current best explanation of corpuscular light propagation seemed to bear no relation to the controversial material laws. The material collision analysis centered around the competing arguments for the primacy of either the quantity of motion or of the *vis viva*. The former is our modern momentum, the product of mass and velocity. And the latter, the "live force", is product of mass and velocity squared - our modern kinetic energy. It is significant for our present discussion that there existed an alternative explanation for light propagation due to Huygens that did not enjoy favor at this time. Although Maupertuis was familiar with Huygens writings he was perfectly comfortable in subscribing to the prevailing opinion and did not feel compelled to seriously contemplate the notion of light as a wave.

Now according to the corpuscular theory, light was understood to travel at a higher speed in a denser medium. Newton wished to explain this higher speed as resulting from an attraction to the denser medium, although as I have already mentioned Maupertuis was suspicious of this force concept, and probably more so when applied to these mysterious particles of light where a mechanical explanation for its origin would be even more doubtful. This led him

$$\frac{m.\,CR\,dCR}{\sqrt{(AC^2+CR^2)}} + \frac{n.\,DR\,dDR}{\sqrt{(BD^2+DR^2)}} = o.$$

Mais, CD étant conſtant, on a $dCR = -dDR$. On a donc

$$\frac{m.CR}{AR} - \frac{n.DR}{BR} = o. \; \& \; \frac{CR}{AR} : \frac{DR}{BR} :: n : m.$$

c'eſt-à-dire, *le ſinus d'incidence, au ſinus de réfraction, en raiſon renverſée de la viteſſe qu'a la lumiere dans chaque milieu.*

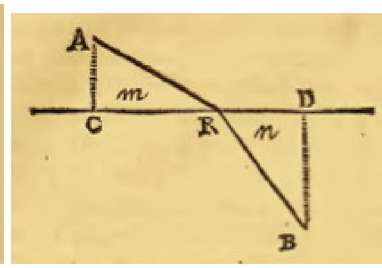

Figure 1: Maupertuis' geometrical derivation of the law of refraction. He assumes that the indices of refraction m and n are proportional to the speed of propagation of the light particle, contrary to the Huygen wave picture in which the speed of the wave is inversely proportional to the speed of the wave. From (*2*)

to conclude that Fermat's principle of shortest time for light travel time was incorrect and he sought an alternative explanation for the law of refraction. In his 1744 article in the Memoires of the Royal Academy of Science, "The accordance of two different laws of nature that had until now seemed incompatible", he has a simple, elegant mathematical demonstration that the law of refraction follows from the minimization of the sum of the products of the (spurious) velocity and distance traversed. (*2*) The same sum of products is minimized for the other two laws governing the motion of light: straight line motion and the law of reflection. It is these three laws that he is unifying in his treatise.

1

He clearly borrowed from the accepted knowledge resources of the time that the particle mass should also come into play since he defines the action as the product of mass, velocity and distance. (He points out that since refraction involves only one particle the mass is a common factor that can be ignored.) But he did not propose to subsume material particles under this general principle of least action until two years later in his Recherche des lois du mouvement (Research on the laws of motion).

There is a substantial irony here, one that is not uncommon in the history of science. Maupertuis rejected the wave picture, leading him to introduce an entirely new physical quantity – the action – and a related law, the principle of least action. Thus we have here the introduction of a new physical quantity based on a spurious assumption. Maupertuis own detailed illustrations of the new principle demanded a questionable and imprecise implementation. The first precise mathematical reformulation was provided by one of historys most gifted mathematicians and Maupertuis colleague at the Berlin Academy, Leonhard Euler. And then, first with contributions from the young Joseph Louis Lagrange and then the 19th century greats William Rowan Hamilton and Carl Gustav Jacob Jacobi the notion of least action evolved into a foundational principle of classical mechanics, in part fulfilling Maupertuis dream of eliminating the force concept from the basic vocabulary of physics at least among the conoscenti. Maupertuis expresses his own doubts regarding both the necessity and meaning of the notion of force in the following passage from his Essay on Cosmology: (*3*) “Motive force, the power a body in motion has to move others, are words invented to substitute for our knowledge, and only signify the results of Phenomena. Only habit keeps us from recognizing what is marvelous about the communication of motion”[1]

I want to say a word here about Maupertuis motivation. He does not receive today the attention he deserves, not so much because of the imprecise formulation of his principle, but because of his metaphysics. (The scientists among us are doubtless aware that this is still largely a pejorative term in our profession.) But within his philosophical system it was logical. He had doubts of human mental capabilities and that without some control the world would digress into chaos. Thus a fundamental principle had to be in effect to regulate the multiplicity of possibilities. Hence, he maintained that his newly found principle was a proof of the existence of god! This was the view he promoted in his popular Essay on cosmology, and then amplified in a 1758 commentary. “In refusing the supposed distinction of mathematical necessity to all of these laws [of Descartes, Leibniz, Huygens, and Newton], one discovers in them another even more precious; namely, evidence of choice of an intelligent and free being. They carry the imprint of the wisdom and power of the creator.”[2]

I cant resist relaying to you an anecdote recently told me by Mary Terall, the author of a wonderful recent biography of Maupertuis. (*4*) She writes “I recall an exchange many, many years ago, when I was just beginning to work on Maupertuis as a grad student, with Murray Gell-Mann, who proclaimed in no uncertain terms that teleology was boring’ ”. Fortunately, we liberal arts types are capable of a more nuanced view!

Now I come to the second irony. The principle of least action is indeed applicable to light, but light as a wave and not as a particle. And in fact Huygens wave principle coincides with Fermat’s principle when the action is suitably reinterpreted. These shufflings in importance and reinterpretations of principles are typical in the history of science, and examples such as this permit us to understand and follow in detail the dynamics of paradigm shift. The shift is contextually rooted in contemporary shared intellectual resources where competing conceptual systems offer contradictory explanations of physical phenomena. This suggests that we would be well-advised to be aware of the contingent historical nature of our own scientific world view. Fortunately this is a lesson that a few more disciplinary physics programs have learned, and they in turn do value the generalist core text programs that we are advocating in this meeting!

Well, this naturally brings us back to the conundrum that I mentioned at the beginning.

[1] La force motrice, la puissance qua un corps en mouvement, den mouvoir dautres, sont des mots inventes pour suppler nos connaissances, & qui signifient que des rsultats de Phnomnes. La seule habitude nous empeche de sentir tout ce quil y a de merveilleux dans la communication du Mouvement. Translation by Mary Terrall (*4*), with permission

[2] En refusant toutes ces Loix la prtendue prrogative dune ncessit, on y en dcouvre une autre bien plus prcieuse; cest le caractre du choix dun tre intelligent & libre: Cest de porter lempreinte de la sagesse et de la puissance de celui qui les a tablies. Translation by Mary Terrall, (*4*) with permission.

How might we combine Einsteins four-dimensional view of space and time with quantum wave mechanics? Maupertuis offers us a clue in the form of the least action, but with an interpretation that is contrary to his initial insight. The clue is provided by the surprising relation between the least action principle and the behavior of light that was first discovered by Hamilton. It is known as the optical-mechanical analogy. Indeed, this analogy played a singular role in the early development of quantum mechanics after the establishment of relativity theory. It turns out that the historical sequence of discovery could easily have been reversed by first assuming that the optical-mechanical wave is real for material particles. It can be shown that this simple assumption alone leads to both Einsteins special relativity of 1905 and the notion of curved space time. (*1*) It is quite possible that a future quantum theory of gravity will involve an as yet unknown iteration of Maupertuis original fertile insight.

References and Notes

1. Alexander Blum, Jürgen Renn, Donald Salisbury, Matthias Schemmel, and Kurt Sundermeyer. Classical wave fronts, accelerated observers, and quantum mechanics. *in preparation*, 2012.

2. Pierre Louis de Maupertuis. Accord de différentes loix de la nature qui avoient jusqu'ici paru incompatibles (the accordance of two different laws of nature that had until now seemed incompatible). *Memoires of the Royal Academy of Science*, 1744.

3. Pierre Louis de Maupertuis. Essai de cosmologie (Essay on cosmology). 1750.

4. Mary Terrall. *The man who flattened the earth: Maupertuis and the science of the Enlightenment*. Chicago, 2002.