Geometrodynamics, Hamiltonian constraints, and canonical quantization

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Collaborators

Much of this work has been performed in collaboration with Josep Pons, Jürgen Renn, Larry Shepley, Kurt Sundermeyer

See arXiv:1606.06076 and 1508.01277 for some of this material and references

Overview

1. INTRODUCTION

Introduction

Focus of this talk:

- I will focus mainly on the discussions through the mid 1970's concerning the realization of the diffeomorphism group as a phase space transformation group, and the associated notion of gravitational observable.

Questions to be addressed:

- What were the underlying fundamental principles that determined the attitude of the key players with regard to the principle of general covariance?
- What factors led to the limited communication between the two major approaches?
- Could the lack of attention to the significance of Hamiltonian constraints have impeded progress in the invention of a quantum theory of gravity?

The ADM action principle

2. WHAT IS MORE FUNDAMENTAL? FREE PARTICLE EXAMPLE AND HAMILTON-JACOBI THEORY

The ADM action principle and the free relativistic particle

ADM - quantum action principle. Major input is first order Lagrangian. [Arnowitt & Deser, 1959] "Further, in the quantum theory, it becomes even more necessary to make this choice [of first order form]. This fact has been stressed by Schwinger in his formulation of quantization which we shall employ". p. 745

Relativistic free particle

$$S = \int dt \left(p_a \dot{q}^a - \left(1 + \vec{p}^2\right)^{1/2} \right) = \int dt \left(p_a \dot{q}^a - H(\vec{p}) \right)$$

The free relativistic particle

Under independent constant variations δq^a and δt about a solution, since the Lagrangian is invariant,

$$\delta S = p_a \delta q^a - H(p_a)\delta t$$

Leads to Hamilton-Jacobi equation

$$\frac{\partial S}{\partial t} + H\left(\frac{\partial S}{\partial q^a}\right) = 0.$$

Classical solutions from complete solution $S(\vec{q}, t; \vec{\alpha})$,

$$\frac{\partial S}{\partial \alpha_a} = 0.$$

The ADM introduction of constraints

Arnowitt, Deser and Misner 1962. [Arnowitt *et al.* , 1962]
Promote to relativistic formulation by introducing q^0 on equal footing with spatial position. Write variation of action by amending the mass shell condition using a Lagrange multiplier λ,

$$\delta S = p_\mu \delta q^\mu - \lambda p_\mu p^\mu$$

Observation that this corresponds to parameterizing the particle four-position $q^\mu(\theta)$, but no effort to investigate within the Hamiltonian formalism the realization of the underlying reparameterization group.
Not necessary since the corresponding 'Hamilton-Jacobi' equation already relates the observable, θ-independent variables q^μ!

$$1 + \frac{\partial S}{\partial q^\mu}\frac{\partial S}{\partial q_\mu} = 0.$$

The ADM action principle

Arnowitt and Deser in 1959 were following the lead of their thesis advisor, Schwinger, in placing their fundamental stress on the transition amplitudes between physically observable states - with the phase of the wave function approximated by the Hamilton principal function,

$$\Psi(q^\mu) \approx e^{\frac{i}{\hbar} S(q)}$$

Wheeler shared this view: [Wheeler, 1964b] "No one has found any way to escape the conclusion that geometrodynamics, like particle dynamics, has a quantum character. Therefore, the quantum propagator, not the classical history, is the quantity that must be well defined." p. 242

[Wheeler, 1964a] “Closer look shows that classical dynamics owes its structure to quantum mechanics”, p. 328

And a familiar quote from the weighty Gravitation[Misner *et al.* , 1973] “The Hamilton-Jacobi description of motion: Natural because ratified by the quantum principle”, p. 641

The true Hamilton-Jacobi equation

But this is not the true Hamilton-Jacobi equation. The true equation follows from the variation of the $q^\mu(\theta)$ and of θ about solutions.

The parameterized free particle actually satisfied the conditions of Rosenfeld's case 1 in his 1930 article (though he did not analyze this specific model). Write the Lagrangian with an auxiliary variable $\mathcal{N}$,

$$L = \frac{1}{2\mathcal{N}} \eta_{\mu\nu} \frac{dq^\mu}{d\theta} \frac{dq^\nu}{d\theta} - \frac{1}{2}\mathcal{N}.$$

Then varying about solutions we find (with π the momentum conjugate to N),

$$\delta S = p_\mu \delta q^\mu + \pi \delta \mathcal{N} + \left(-p_\mu \frac{dq^\mu}{d\theta} + L - \pi \frac{d\mathcal{N}}{d\theta} \right) \delta\theta \qquad (1)$$

$$= p_\mu \delta q^\mu + \pi \delta \mathcal{N} - H_{RBD} \delta\theta,$$

where

$$H_{RBD}(q^\mu, \mathcal{N}, p_\nu, \pi, \theta) = p_\mu \frac{dq^\mu}{d\theta} - L + \lambda\pi$$

is the vanishing Rosenfeld-Bergmann-Dirac Hamiltonian.

The true Hamilton-Jacobi equation

So the Hamilton-Jacobi equation is

$$H_{RBD}\left(q^{\mu}, \mathcal{N}, \frac{\partial S}{\partial q^{\mu}}, \frac{\partial S}{\partial \mathcal{N}}, \theta\right) + \frac{\partial S}{\partial \theta} = 0.$$

Since we have the general solution for the free particle we merely need to substitute into the action to get the complete solution. It is

$$S(q^\mu, \mathcal{N}, \theta, \bar{p}^\nu, \bar{\pi}) = \bar{p}_\mu x^\mu + \left(\bar{\pi} - \frac{\theta}{2}(\bar{p}^2 + m^2c^2)\right)\mathcal{N}$$

$$+\frac{1}{2}\left(\bar{p}^2 + m^2c^2\right)\left(\theta \int_0^\theta d\theta_1 \lambda(\theta_1) - \int_0^\theta d\theta_1 \int_0^{\theta_1} d\theta_2 \lambda(\theta_2)\right)$$

$$-\bar{\pi} \int_0^\theta d\theta_1 \lambda(\theta_1)$$

But to recover the particle trajectories one must impose the mass shell condition on the constants $\bar{p}^\mu$ <u>after</u> performing partial derivatives!

Phase space generator

Rosenfeld also showed, as did Bergmann and his collaborators in the early 1950's, that the phase space generator of infinitesimal reparameterizations is the variation (1) evaluated for arbitrary infinitesimal reparameterizations $\theta' = \theta + \epsilon(\theta) = \theta - \mathcal{N}^{-1}\xi(\theta)$. Since this is a symmetry variation the generator vanishes. The (active) variations are $\bar{\delta} q^\mu = p^\mu \xi$, $\bar{p}_\nu = 0$, and $\bar{\delta}\mathcal{N} = \frac{d(\mathcal{N}\xi)}{d\theta}$.

Bergmann's reduced phase space

In 1970 Bergmann [Bergmann, 1970] employed a model similar to the free particle to illustrate how the action of this generator sweeps out equivalence classes in phase space. I will illustrate in 1+1 dimensions.

Thus the gauge trajectories are $q^1 = \hat{q}^1 + \frac{p^1}{p^0}\left(q^0 - \hat{q}^0\right)$, and $p^1 = constant$ for fixed $\hat{q}^1$ and $\hat{q}^0$.

Note that since the general solutions for arbitrary parameterizations are $q^0(\theta) = \bar{q}^0 + f(\theta)p^0$ and $q^1(\theta) = \bar{q}^1 + f(\theta)p^1$ for constant $\bar{q}^0$ and $\bar{q}^1$, both p^1 and $\hat{q}^1$ are independent of θ. They are reparameterization invariants. They also satisfy the Poisson bracket relations $\left\{\hat{q}^1, p_1\right\} = 1$ and $\left\{\hat{q}^1, p^0\right\} = p^1/p^0$.
The $\hat{q}^1$ and p_1 are coordinates of the reduced phase space. This is the algebra of the Newton-Wigner spatial position operator, so this construction actually brings one closer to a four-dimensional formalism.

Bergmann's reduced phase space

Note that these invariants can be obtained through the performance of a finite reparameterization-induced symmetry transformation with finite variable-dependent descriptor:

Choose the gauge $\theta = \hat{q}^0(\theta)$, then find the descriptor that gauge transforms $q^0(\theta)$ to θ, i.e.

$$\theta = \hat{q}^0(\theta) = q^0(\theta) + \xi p^0$$

so the finite descriptor is

$$\xi = \frac{\theta - q^0(\theta)}{p^0},$$

and the gauge transformed spatial variables are

$$\hat{q}^a(\theta) = q^a(\theta) + \frac{\theta - q^0(\theta)}{p^0} p^a.$$

3. BERGMANN-KOMAR HAMILTON-JACOBI THEORY

Bergmann Komar HJ theory

Bergmann began his analysis of the Hamilton-Jacobi formulation of general relativity in 1966 [Bergmann, 1966]. He assumed from the start that the appropriate metric variables were the spatial metric components, and that one could work exclusively in the constrained phase space. He was able to prove that the Hamilton principle function could contain no explicit dependence on the arbitrary spacetime coordinates. This led him to the declaration that ”time is frozen”.

Note that this is consistent with the particle example in which S on shell is independent of θ.

Bergmann Komar HJ theory

Both Bergmann and Komar followed up on this initial analysis which described the reduced phase space in general relativity, with the true two degrees of freedom of the classical gravitational field. These variables were canonical, and they did generate phase transformations between equivalence classes. [Komar, 1967] [Komar, 1968] [Komar, 1970a] [Bergmann *et al.* , 1970] [Komar, 1971] [Bergmann, 1971][Bergmann, 1973a][Bergmann, 1973b][Komar, 1980]

Neither Bergmann nor Komar addressed the question, in the context of Hamilton-Jacobi theory, of whether intrinsically determined coordinates could be introduced which would describe time evolution. This is a surprise since they were the first to introduce this idea a decade earlier!

Bergmann Komar HJ theory

A quotes from Komar 1970 [Komar, 1970b]

"In view of the fact that that the observables of a theory, when expressed in terms of their operator aspect, must generate canonical transformations compatible with the equations of motion, it follows that the observables of the general theory of relativity must must commute with the generators of the Einstein group! It therefore appears to be impossible to employ the observable dynamical variables of the general theory to provide a realization of the Einstein group. There being no other distinguished spacetime symmetry group available, it would appear that the possibility of constructing a unique quantum theory of gravitation is placed in jeopardy."

"Rather than exploring this point further, in this section we propose to impose a preferred spacetime symmetry"

The specialization is to asymptoticaly flat spacetimes with a preferred Bondi-Metzner-Sachs asymptotic symmetry.

ARNOWITT-DESER-MISNER ‘INTRINSIC COORDINATES’

The ADM approach

Arnowitt, Deser, and Misner, following Schwinger’s lead in quantum electrodynamics, proposed that the diffeomorphism-invariant degrees of freedom of general relativity could be attained through the imposition of coordinate conditions. [Arnowitt & Deser, 1959, Arnowitt *et al.* , 1959b, Arnowitt *et al.* , 1959a, Arnowitt *et al.* , 1960d, Arnowitt *et al.* , 1960e, Arnowitt *et al.* , 1960f, Arnowitt *et al.* , 1960b, Arnowitt *et al.* , 1960a, Arnowitt *et al.* , 1960c, Arnowitt *et al.* , 1961b, Arnowitt *et al.* , 1961a, Arnowitt *et al.* , 1962]

Asymptotic coordinate conditions

Similarly to the later Komar-Bergmann approach, they considered asymptotically flat spacetimes. Their proposal, following Arnowitt and Deser's initial discussion of linearized gravity, was to identify the traceless, transverse components of the three metric as representing the true dynamical degrees of freedom. They showed that this choice could be implemented through the coordinate conditions

$$x^0 = -\nabla^{-2}(\pi^T + \nabla^2 \pi^L),$$

and

$$x^a = g_a - \frac{1}{4}\nabla^{-2} g^T_{,a}.$$

Perferred intrinsic coodinates

They insisted that there did indeed exist preferred global frames of reference, and these were determined by their requirement that the Hamiltonian must not depend on their intrinsic time x^0. They showed that their intrinsic choice satisfied this requirement, and that furthermore permissible canonical transformations corresponding to changes in intrinsic coordinates should be limited by this time independence requirement.

Curiously(?), ADM never made any reference to Bergmann and Komar's introduction of intrinsic coordinates - starting in 1958 - nor to the Bergmann school's analysis of constrained Hamiltonian dynamics. [Komar, 1958]

Nor did Bergmann and Komar later extensively cite ADM!

KOMAR BERGMANN INTRINSIC COORDINATES

Bergmann and Komar launched a program in 1958 for correlating the dynamically determined temporal development and spatial position with the behavior of the gravitational field itself. [Bergmann & Komar, 1960, Bergmann, 1961b, Bergmann, 1961a, Bergmann & Komar, 1962]. This was actually the first suggested use of intrinsic coordinates. They insisted in the vacuum case that the intrinsic coordinates be local spacetime scalar functionals of the metric. They were able to show that the Weyl scalars could be used for this purpose. Furthermore, they showed that these scalars could be written in terms of the three metric and conjugate momenta. [Bergmann & Komar, 1960]

A thorough discussion of the proposed use of intrinsic coordinates can be found in Bergmann's 1962 Handbuch der Physik article [Bergmann, 1962]

Why did they abandon this program? I suggest that it has to do with the breakthrough that Dirac made in 1958 [Dirac, 1958] in simplifying the primary constraints of general relativity. There are two significant elements of this story. The first is that Dirac choose to consider infinitesimal coordinate transformation as either tangent to an initial constant time hypersurface, or as along the direction perpendicular to the surface,

$$x'^{\mu} = x^{\mu} + n^{\mu}\xi^{0} + \delta^{\mu}_{a}\xi^{a},$$

where $n^{\mu} = (N^{-1}, -N^{-1}N^{a})$ is the perpendicular, N is the lapse, and N^{a} is the shift. Secondly, Dirac isolated those canonical variables, including those involving time derivatives, that did not vary under four-dimensional diffeomorphisms that did not alter the chosen foliation. The outcome for Bergmann was that, contrary to his initial Hamiltonian formulation with Anderson in 1951 [Anderson & Bergmann, 1951], one is justified in dropping the lapse and shift as canonical variables.

However, Bergmann and Komar did in 1972 [Bergmann & Komar, 1972] give a group theoretical explanation for the Dirac Poisson algebra that resulted from his infinitesimal diffeomorphism decomposition. They showed that one was not realizing in phase space the original diffeomorphism group, but rather a diffeomorphism-induced phase space transformation group. This group has come to be called the Bergmann-Komar group.

AN INTRINSIC HAMILTON-JACOBI APPROACH

The generator of infinitesimal transformations

This is where I and my collaborators enter the picture. [Pons *et al.* , 1997, Pons *et al.* , 2000c, Pons *et al.* , 2000a, Pons *et al.* , 2000b, Pons & Salisbury, 2005][Pons *et al.* , 2009a, Pons *et al.* , 2009b, Pons *et al.* , 2010] We have shown for several models, including classical Einstein-Yang-Mills and also employing Ashtekar variables, that the generator of the full four-dimensional diffeomorphism-induced group is of the form

$$\mathcal{G}_\xi(t) = P_\mu \dot{\xi}^\mu + (\mathcal{H}_\mu + \int d^3x' \int d^3x'' N^{\rho'} C^{\nu''}_{\mu\rho'} P_{\nu''})\xi^\mu .$$

where

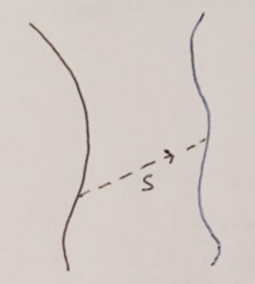
$$\{\mathcal{H}_\mu(x), \mathcal{H}_\rho(x')\} = C^{\nu''}_{\mu\rho'} \left[g_{ab}\right] \mathcal{H}_{\nu''}$$

Time evolution versus diffeomorphisms

The evolution in time is generated by

$$H = \int d^3x \, (N\mathcal{H}_0 + N^a\mathcal{H}_a + \lambda_\mu P^\mu) \, .$$

The finite diffeomorphism generator $\exp\left(s \int d^3x \, \mathcal{G}_\epsilon(t)\right)$ transforms solutions into new solutions.



Enlargement of phase space

Note that the lapse function N and shift N^a must be retained as canonical variables.

Note also that contrary to popular belief, the Hamiltonian formulation does not fix a time foliation. New foliations result in new multipliers λ^μ and new Hamiltonians as a consequence of the time dependence of the Hamiltonian.

4. CLASSICAL INTRINSIC DYNAMICS AND NATURAL WHEELER DEWITT EQUATIONS

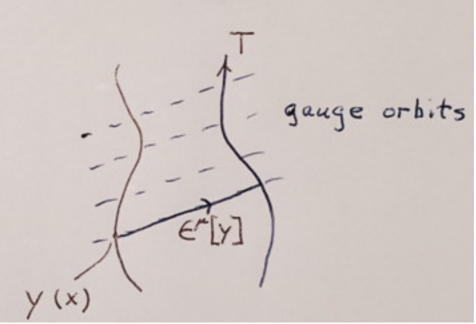
The implementation of Rovelli's partial variable program

Now that we have the full diffeomorphism group at our disposal, we can employ it to establish correlations between partial variables. One possible implementation, in principle, is to locate temporal and spatial landmarks by referring to curvature even in the vacuum case. There are of course many more possibilities when matter is present. We will employ these landmarks as "intrinsic" coordinates. Such coordinates must be formed from spacetime scalars. Thus we choose $X^{\mu}[g_{ab}, p^{ab}]$.

In the vacuum case we propose the use of the four Weyl curvature scalars, as originally suggested by Komar in the 1950's. They are quadratic and cubic in the Weyl tensor. Bergmann and Komar showed in 1960 that they are expressible solely in terms of the three metric and its conjugate momenta.

Intrinsic coordinate gauge conditions

We choose intrinsic coordinates through the gauge conditions $x^\mu = X^\mu[g_{ab}, p^{ab}]$. Given any solution trajectory in phase space we can then determine the phase space dependent finite descriptors $\epsilon^\mu[g_{ab}, p^{ab}] := \epsilon^\mu[y]$ that will gauge transform these solutions to those that satisfy the gauge conditions.



The explicit construction of evolving constants of the motion

This construction yields Taylor expansions in the coordinates x^μ - now themselves diffeomorphism invariants. The coefficients in the Taylor expansions are functionals of g_{ab} and p^{ab} that are explicitly diffeomorphism invariants. This applies also to the invariant lapse and shift.

$$\mathcal{I}_\phi = \sum_{n_\mu=0}^{\infty} \frac{1}{n_0!\, n_1!\, n_2!\, n_3!} (x^0)^{n_0} (x^1)^{n_1} (x^2)^{n_2} (x^3)^{n_3}\, \mathcal{C}_{n_0,n_1,n_2,n_3}[g_{ab}, p^{ab}]$$

Kuchar-inspired canonical transformations

Canonical transformations can be carried out to new canonical variables including X^μ and canonical conjugates π_μ - but without imposing gauge conditions. The theory in terms of these new variables is still fully diffeomorphism covariant - with corresponding Hamiltonian constraints. Each choice yields a new form for the constraints and a new Wheeler-DeWitt equation with a corresponding "natural" choice of temporal and spatial partial variables.

This "natural" choice is the one that results through the solutions of the Wheeler-DeWitt equation.

Fully relative general relativity

Claim: The range of intrinsic coordinates is coincident with the set of coordinates obtained under arbitrary coordinates transformations

Corollary: For every choice of coordinate chart there is a corresponding choice of intrinsic coordinates.

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