Rosenfeld, Bergmann, Dirac and the Invention of Constrained Hamiltonian Dynamics

Don Salisbury

Austin College

Marcel Grossmann 11

Berlin, July 2006

Plan of Talk

1. Brief biographical background of Rosenfeld and Bergmann
2. Singular Lagrangian prehistory
3. Rosenfeld's formal constraint analysis
4. Rosenfeld's gravitational model

1 - Brief Rosenfeld biography

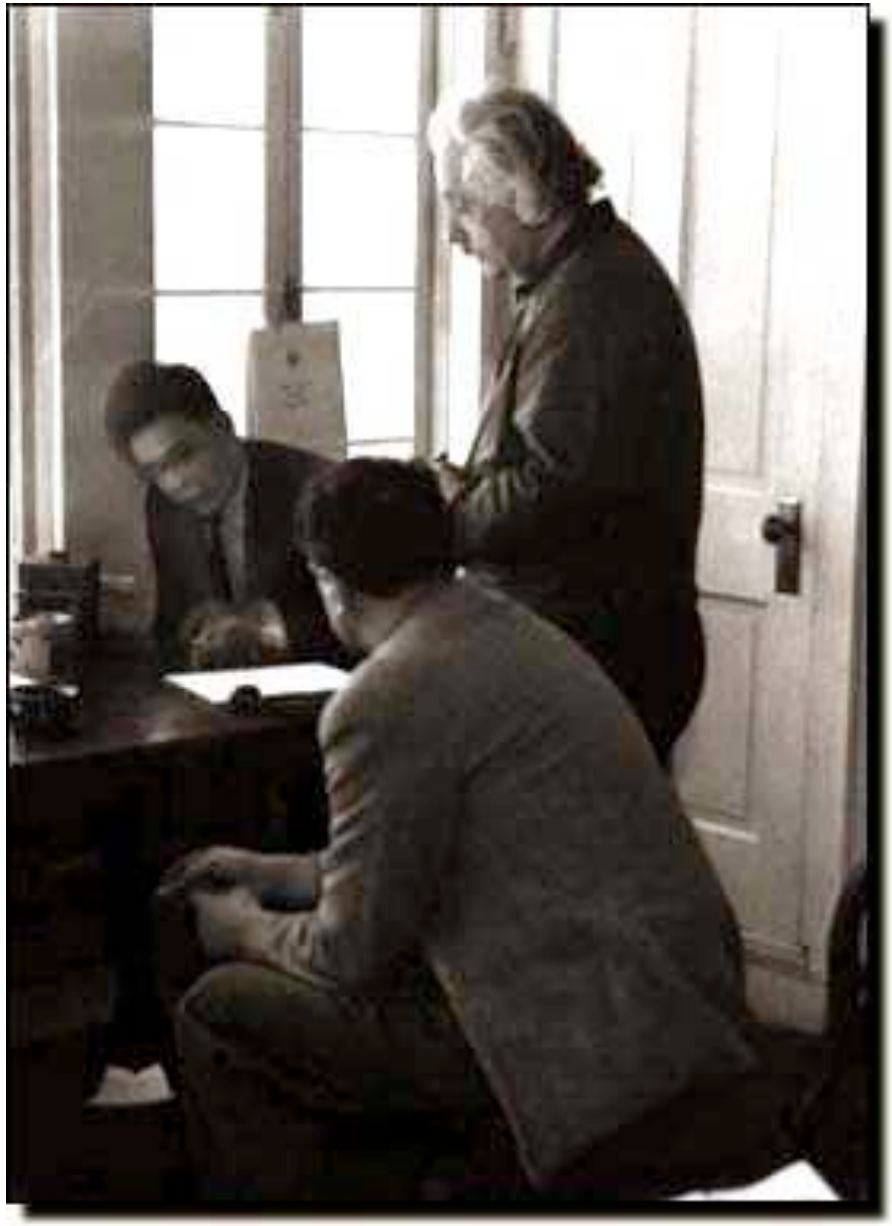
- Born Belgium 1904
- Doctorate Liege 1926
- Research in Paris, Göttingen, Zurich 1926-1930
- Taught theoretical physics at Liege, Utrecht, Manchester, Copenhagen 1940-1974
- Collaborators and correspondents: Bohr, Pauli, de Broglie, Dirac, Heisenberg, Infeld, Klein …
- Died October 1974

Heisenberg and Rosenfeld

Rosenfeld (right, standing)
at 1933 Solvay Meeting

Brief Bergmann biography

- Born Berlin-Charlottenburg 1915
- Mother Dr. Emmy Bergmann moved with children to Freiburg 1922 - she and sister emigrated to Israel 1935
- Father Dr. Max Bergmann 1921 - 1933 head of Institut für Lederforschung, Dresden (now Max Bergmann Zentrum für Biomaterialen)
- Prague, Charles University degree 1936
- Einstein Assistant 1936 - 1941: unified field theory
- Syracuse University 1947 - 1982
- Died October 2002



2 - Singular Lagrangian prehistory

Emmy Noether's second theorem in "Invariante Invariationsproblem", 1918

$$\bar{\delta} L \equiv -\frac{\partial}{\partial x^{\mu}}\left(L \xi^{\mu}\right), \quad \text{variation (Lie drivative) under } x'^{\mu} = x^{\mu} + \xi^{\mu}(x)$$

Suppose that the variations of the dynamical variables $y_A(x)$ are of the form

$$\bar{\delta} y_A = {}^{0}f_{Ai}(x, y, \ldots)\xi^{i} + {}^{1}f_{Ai}^{\mu}(x, y, \ldots)\xi^{i}_{,\mu}$$

Let $L^A = 0$ represent the Euler-Lagrange equations, then for vanishing ξ^{μ} on the action integration boundary

$$L^A \; {}^{0}f_{Ai} - \frac{\partial}{\partial x^{\mu}}\left(L^A \; {}^{1}f_{Ai}^{\mu}\right) \equiv 0$$

Contracted Bianchi identities in general relativity

Obstacles on the road to quantum electrodynamics

Free electromagnetic field Lagrangian invariant under

$$\delta A_\mu = \xi_{,\mu} \Rightarrow 0 \equiv \frac{\partial}{\partial x^\mu}\left(L^A \ {}^1f^{\mu}_{Ai}\right) = F^{\mu\nu}_{,\mu\nu}$$

Proposals for dealing with vanishing momentum $\pi^0 = \partial L / \partial A_{,0} = 0$

Heisenberg/Pauli formalism (1929-1930)

- Add non-gauge invariant term to Lagrangian
- Or set A_0 *= constant* (destroys manifest Lorentz invariance)

Pauli: "Ich warne Neugierige"

Rosenfeld's debt to Pauli: "*As I was investigating these relations in the especially instructive example of gravitation theory, Professor Pauli helpfully indicated to me the principles of a simpler and more natural manner of applying the Hamiltionian procedure in the presence of identities*"

3 - Rosenfeld's formal constraint analysis in "On the quantization of wave fields", Annalen der Physik 1930

Considers Lagrangians quadratic in field derivatives

$$L = \frac{1}{2}\Lambda^{A\mu B\nu}(x,y)\,y_{A,\mu}\,y_{B,\nu} + \ldots$$

and assumes invariance (including possible surface term) under variations of the form

$$\bar{\delta} y_A = {}^0f_{Ai}(x,y)\xi^i + {}^1f_{Ai}^{\mu}(x,y)\xi^i_{,\mu} - y_{A,\mu}\xi^\mu = \delta y_A - y_{A,\mu}\xi^\mu$$

3A: Deduces from the Noether identity, setting to zero coefficent of $\xi^i{}_{,000}$:

1) $\pi^A(y,\dot{y})\, {}^1f_{Ai}^0(y) = \Lambda^{A0B\nu}(y,\dot{y})\,y_{B,\nu}\; {}^1f_{Ai}^0(y) \equiv 0$ *i* primary constraints

2) $\Rightarrow \Lambda^{A0B0}(y)\; {}^1f_{Ai}^0(y) \equiv 0$ Legendre matrix has *i* null vectors

3) $\pi^A = \Lambda^{A0B0}(y)\dot{y}_B + d^A(y) \Rightarrow$ can add $\lambda^i(x)\,{}^1f_{Ai}^0(y)$ to $\dot{y}_A$

These results were obtained independently in 1949 by Bergmann in "Non-linear field theories", employing the arguments!

3B: Rosenfeld supposes that solutions have been found for all of the velocities in terms of momentum, and these solutions $v_A(y,\pi)$ are used to construct the canonical Hamiltonian $H_0 = \pi^A v_A - L(y,v)$

Then he takes his Hamiltonian to be $H = H_0 + \lambda^i(x,y) {}^1f^0_{Ai}(y)\pi^A$

Bergmann and Brunings obtain a similar formal result in 1949. Since they work in parameterized formalism their Hamiltonian vanishes. Bergmann, Penfield, Schiller, and Zatkis in 1950 invent algorithm for solving for $v_A(y,\pi)$. Rosenfeld never addresses this question.

Dirac deduces similar formal result in 1949 Vancouver lectures. Employs independent variations in configuration-velocity-momentum space, distinguishing between weak and strong equalities. Work published in 1950.

3C: Rosenfeld finds general canonical expression for generators $C(\xi)$ of symmetry transformations $\bar{\delta} y_A$. They consist of a geometric transformation δ plus a transport term

$$C(\xi) = \int d^3x \left(\pi^A \delta y_A - \left(\pi^A y_{A,\mu} - \delta_\mu^0 L \right) \delta x^\mu \right)$$

geometric transformation   transport term

Rosenfeld writes $$C(\xi) = \int d^3x \left({}^0A_i \xi^i + {}^1A_i \frac{\partial \xi^i}{\partial t} \right)$$

Rosenfeld proves that $C(\xi)$ generates the correct variations of y_A AND π^B! In particular he shows without comment that no higher time derivatives of ξ^i appear in variations.

Dirac never addresses realizability of symmetry group as canonical transformations

In 1951 Bergmann and Anderson express $C(\xi)$ in second form. This is after discovery of Rosenfeld paper. *"This examination … is in some respects similar to the results obtained by L. Rosenfeld"*.

3D: Rosenfeld uses the Noether identity to show that $C(\xi)$ is a constant of the motion. Furthermore, he shows that the ${}^{0}A_i$ are the primary constraints. Then he develops a recursion relation, showing in particular that ${}^{1}A_i$ are the time derivatives of the primary constraints, i.e., they are secondary constraints!

Bergmann and Anderson applied the same arguments to get the same results, but they have been given credit for the discovery - and for the terminology. Their significant contribution is the derivation of Poisson bracket relations among the primary, secondary, … constraints.

4. Rosenfeld’s dynamical model - gravitation with a charged spinorial Dirac field source

Origins of the model

- Weyl/Fock coupling of Dirac field with gravity - 1929
- Tetrads and Weyl’s reinterpretation of gauge symmetry
  See analyses by Scholz (physics/0409158) and Straumann (hep-ph/0509116)

Rosenfeld’s vacuum gravity contribution to the Lagrangian

$$L_{Rosenfeld} = \sqrt{-{}^4 g}\,{}^4R + \frac{\partial}{\partial x^\mu}\left(2\sqrt{-{}^4 g}\,E_I^\mu E_J^\nu \omega_\nu^{IJ}\right)$$

tetrad

Ricci rotation coefficient

Missed opportunity: if Rosenfeld had used tetrads adapted to the spacetime foliation he would have achieved the definitive simplification of the gravitational Lagrangian published by Dirac in 1958

Take the timelike tetrad to be orthogonal to the spacetime foliation, then the remaining triads are tangential, and

$$E_0^\mu = \left(N^{-1}, -N^{-1}N^a\right)$$

The Rosenfeld Lagrangian differs from the real part of the Lagrangian in the Ashtekar formalism only by a spatial divergence. (See Salisbury, Shepley, and Pons, PRD62, 064026 (2000)). So it does not contain time derivatives of $E_0{}^\mu$.

As noted in the analagous model of Dirac, it is then possible to eliminate $E_0{}^\mu$ entirely from the canonical formalism - ignoring some significant problems with implementing symmetry transformations!