Leon Rosenfeld invented indeterminate gauge dynamics in 1930

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Plan of Talk

1. Gauge symmetry historical background
2. Example of the canonical quantization of a fully deterministic system
3. Examples of indeterminate gauge dynamics
4. Obstacles to canonical quantization of electrodynamics
5. Heisenberg/Pauli and Fock approaches to quantum electrodynamics
6. Leon Rosenfeld's 1930 paper
7. The impact of Rosenfeld's work

I - Gauge symmetry historical background

- Maxwell’s electromagnetism
- Einstein’s general theory of relativity
- Weyl’s gauge theory
- Noether’s second theorem

II - Example of the canonical quantization of a fully deterministic system

Consider a free relativistic particle with the position x a function of the time t. Then if the position and velocity are specified at time t_0, the positions at all future times is uniquely determined.

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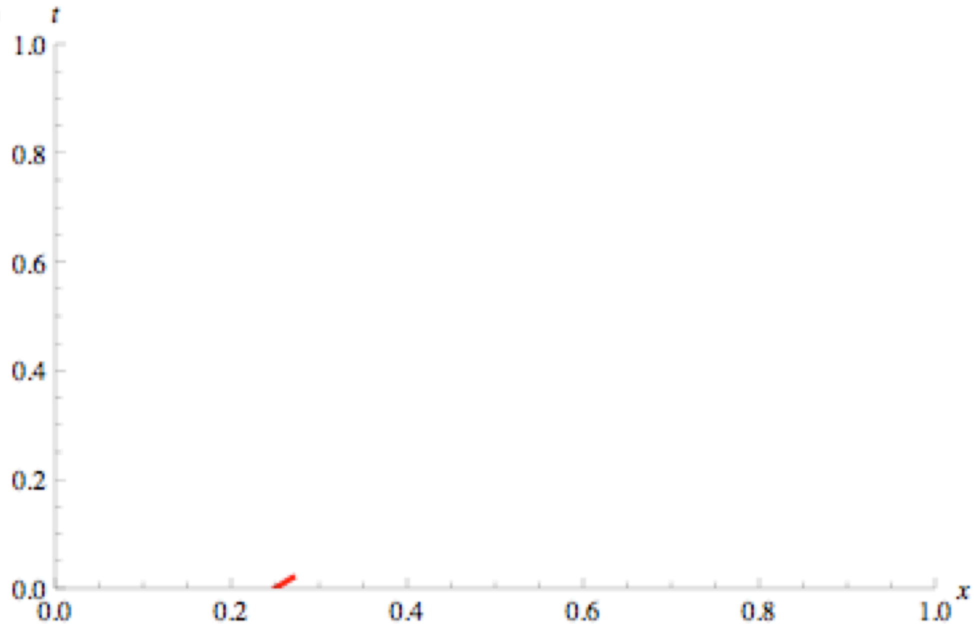

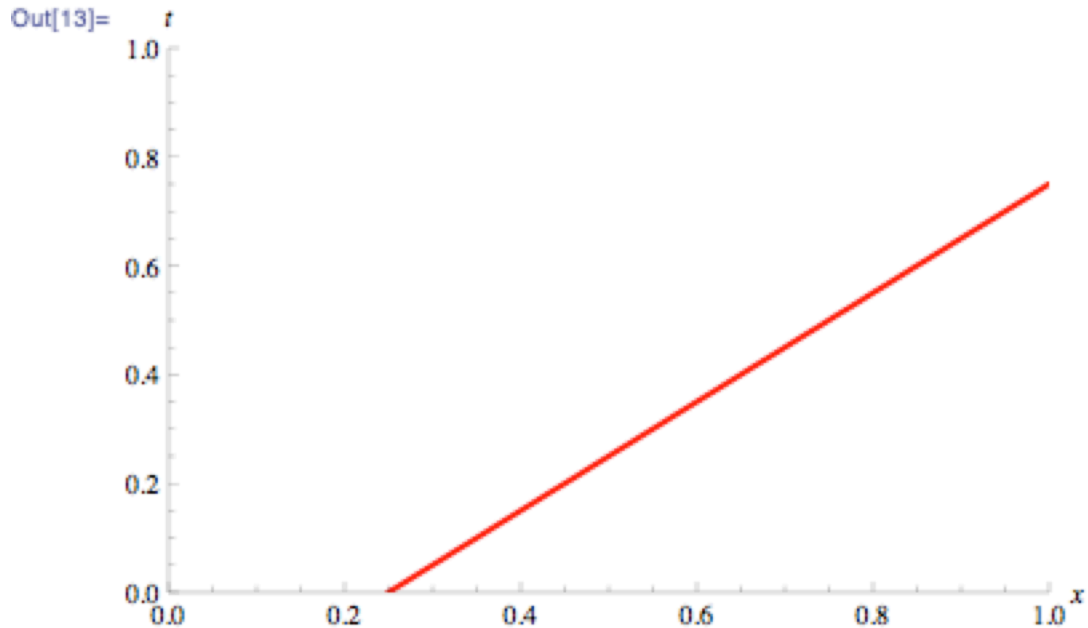
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Canonical Hamiltonian formalism

Classical Lagrangian formulation. Spatial position *$x^a(t)$*.

$$L = -mc^2 \left(1 - \frac{v^2}{c^2}\right)^{1/2} \qquad v^a := \frac{dx^a}{dt} =: \dot{x}^a$$

Lagrangian equations of motion

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}^a} = 0$$

Conjugate momentum

$$p^a = \frac{\partial L}{\partial \dot{x}^a} = m\left(1 - \frac{v^2}{c^2}\right)^{-1/2} \dot{x}^a \qquad \Rightarrow \dot{x}^a = \frac{p^a c^2}{(m^2c^4 + p^2c^2)^{1/2}}$$

Hamiltonian

$$H = p_a \dot{x}^a - L = \left(m^2 c^4 + p^2 c^2\right)^{1/2}$$

First-order Hamiltonian equations of motion

$$\dot{x}^a = \frac{\partial H}{\partial p_a} = \frac{p^a c^2}{(m^2c^4 + p^2c^2)^{1/2}} \qquad \dot{p}^a = -\frac{\partial H}{\partial x_a} = 0$$

Symmetry

Lagrangian equations of motion have same form under a Lorentz transformation to a new inertial frame

$$x'^{\mu} = x^{\mu} + \epsilon^{\mu}{}_{\nu} x^{\nu}$$

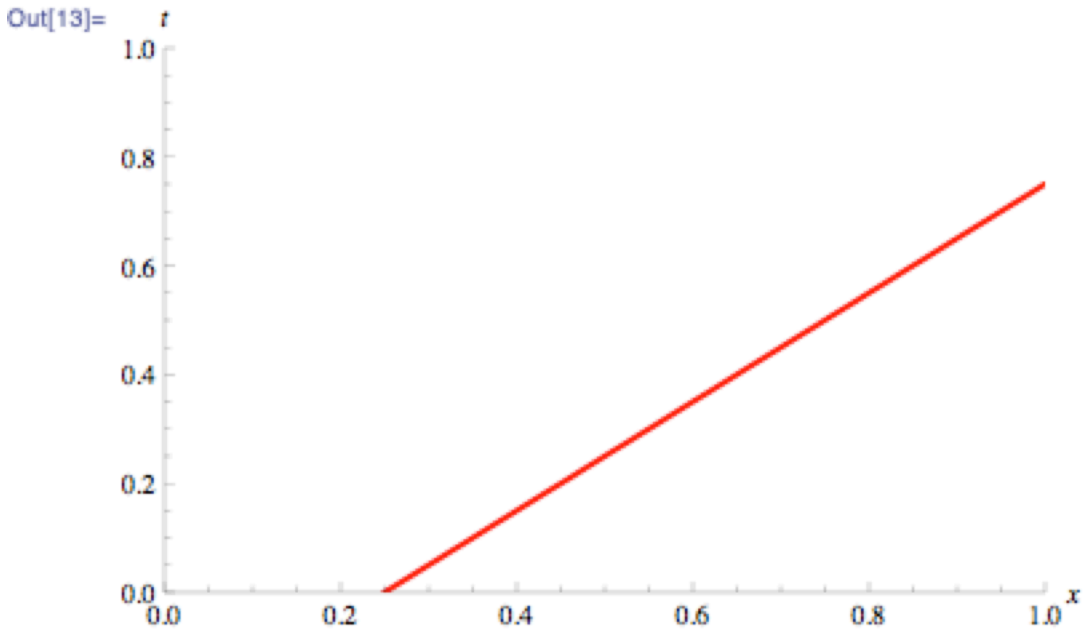
Lagrangian has same form since

$$L dt = -m \left(c^2 dt^2 - dx^2 - dy^2 - dz^2 \right)^{1/2} = -m ds = L' dt'$$

Noether's first theorem implies a related conserved quantity - the generator of Lorentz transformations (including angular momentum)

$$\delta \int_{t_1}^{t_2} L\, dt = - \int_{t_1}^{t_2} \frac{d}{dt} \frac{\partial L}{\partial \dot{x}^a} \delta x^a + \frac{\partial L}{\partial \dot{x}^a} \delta x^a |_{t_1}^{t_2} \equiv - \frac{\epsilon^0{}_a}{c} L x^a |_{t_1}^{t_2}$$

The resulting generator in the Hamiltonian formalism transforms solutions into new solutions



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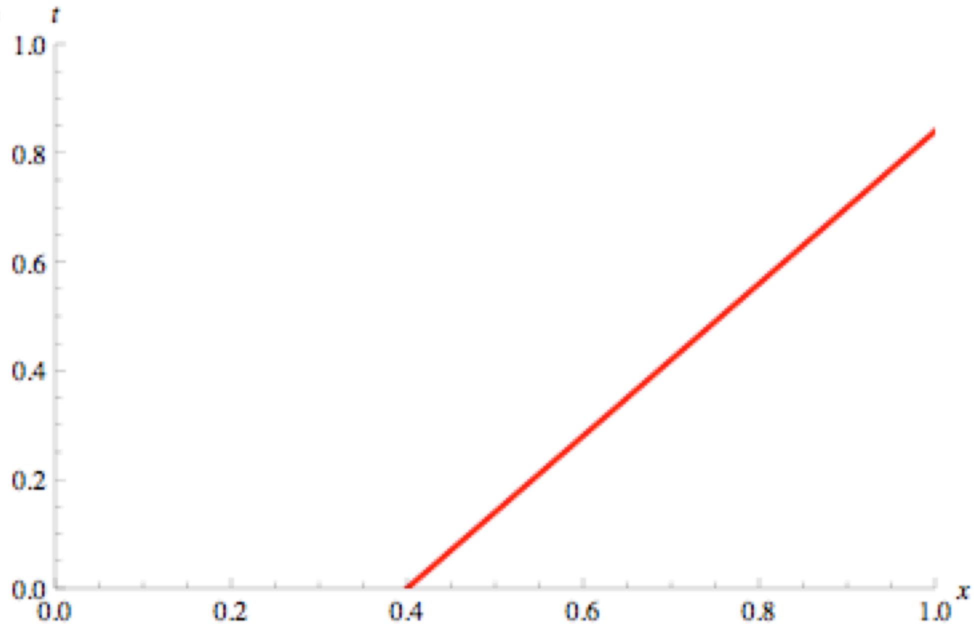

Canonical quantization. Poisson bracket

$$\{x^a, p_b\} := \Sigma_c \left(\frac{\partial x^a}{\partial x^c}\frac{\partial p_b}{\partial p_c} - \frac{\partial x^a}{\partial p^c}\frac{\partial p_b}{\partial x_c} \right) = \delta^a_b \rightarrow [x^a, p_b] = i\hbar\delta^a_b$$

Physical quantities become operators. For example, the *1* component of the angular momentum operator is

$$L^1 = x^2 \frac{\hbar}{i}\frac{\partial}{\partial x^3} - x^3 \frac{\hbar}{i}\frac{\partial}{\partial x^2}$$

III - Examples of Indeterministic Dynamics

Example 1: Parameterized relativistic free particle

Take particle postion and time to be functions of a parameter θ, $x^\mu(\theta)$

Then Action becomes $S = \int_{\theta_1}^{\theta_2} L \, d\theta$

where the Lagrangian is

$$L = -mc^2 \left((\dot{x}^0)^2 - (\dot{x}^1)^2 - (\dot{x}^2)^2 - (\dot{x}^3)^2 \right)^{1/2} = -mc^2 \left(-\dot{x}^2 \right)^{1/2}$$

The Lagrangian equations of motion are covariant under arbitrary reparameterizations

$$\theta' = \theta'(\theta)$$

Thus one can arbitrarily reparameterize a solution to obtain a solution with a different functional form. This means that there is no unique evolution from given initial values

$$x^\mu(\theta_1), \frac{dx^\mu(\theta_1)}{d\theta}$$

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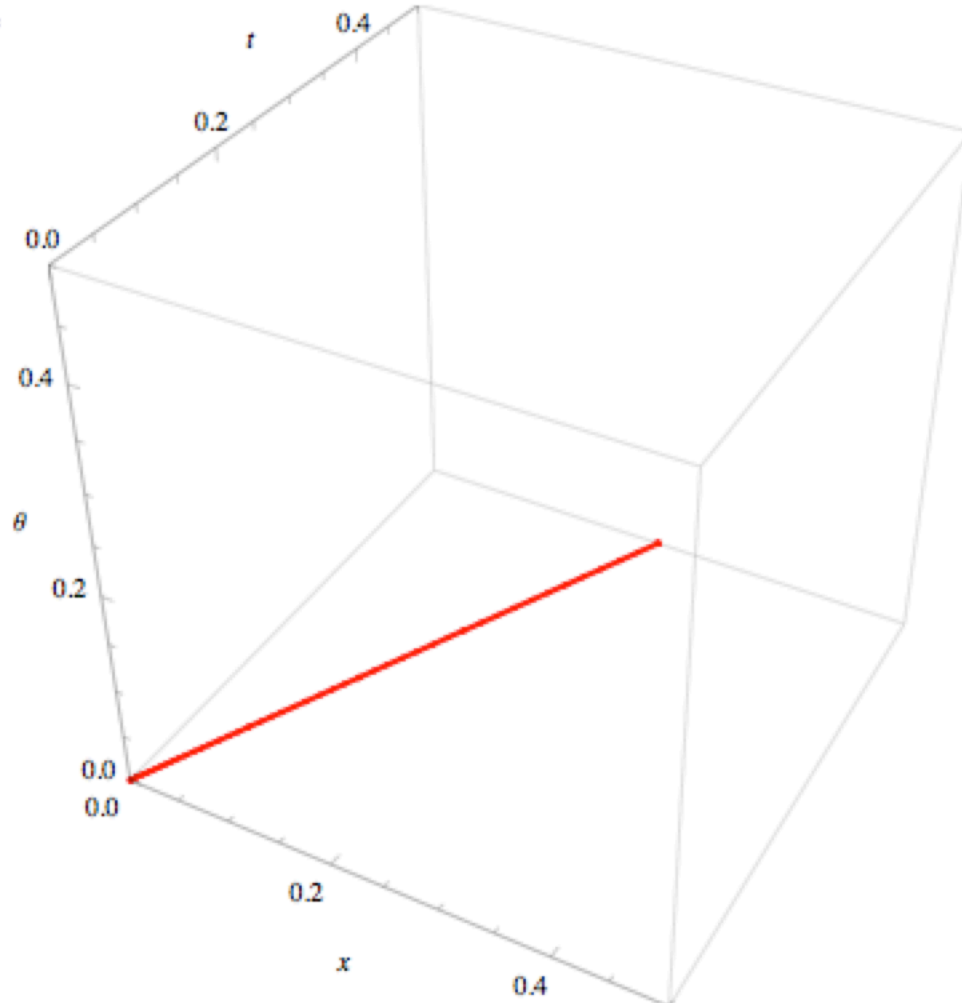

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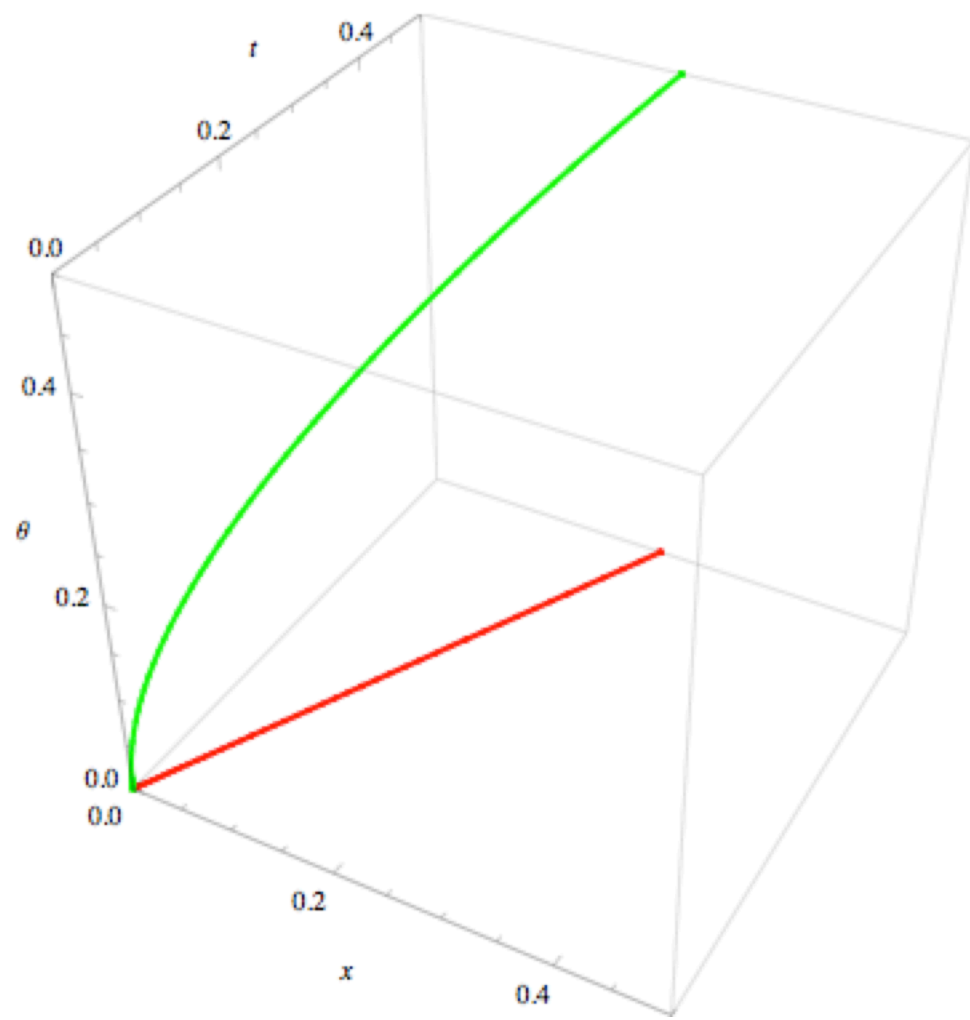

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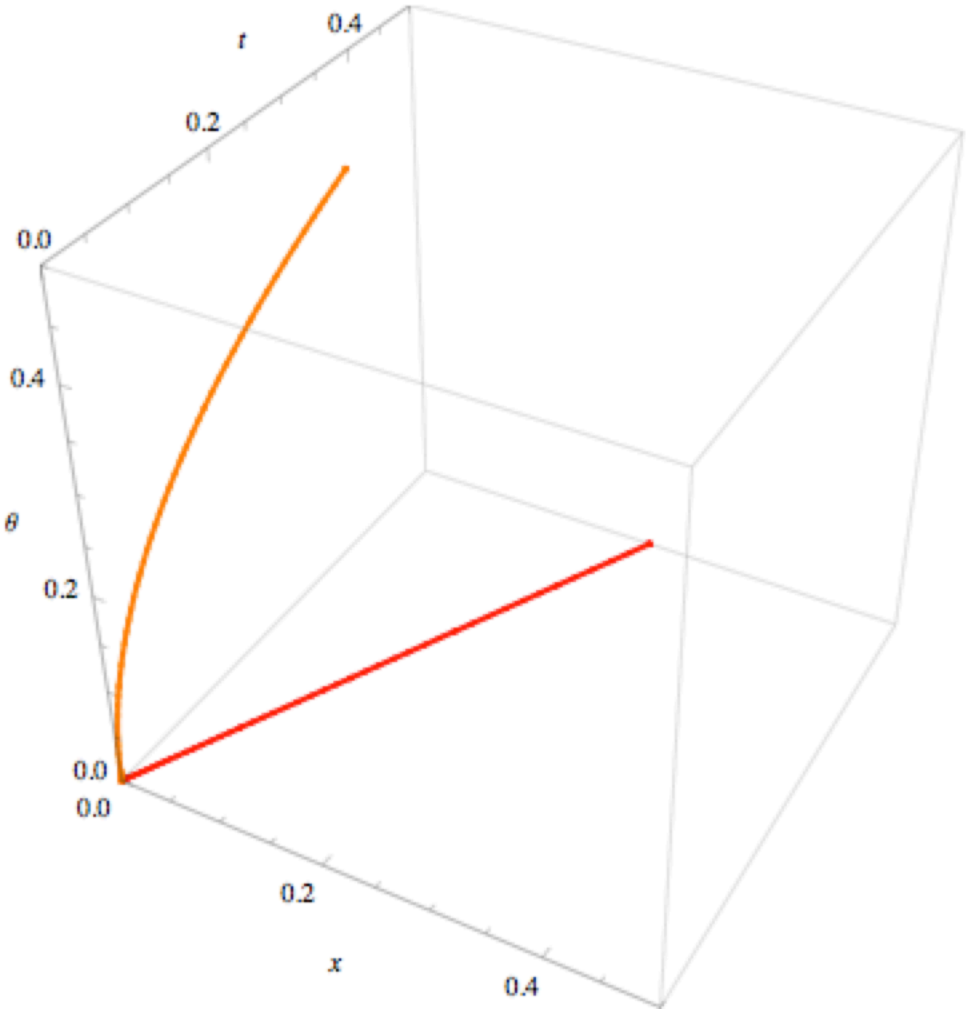

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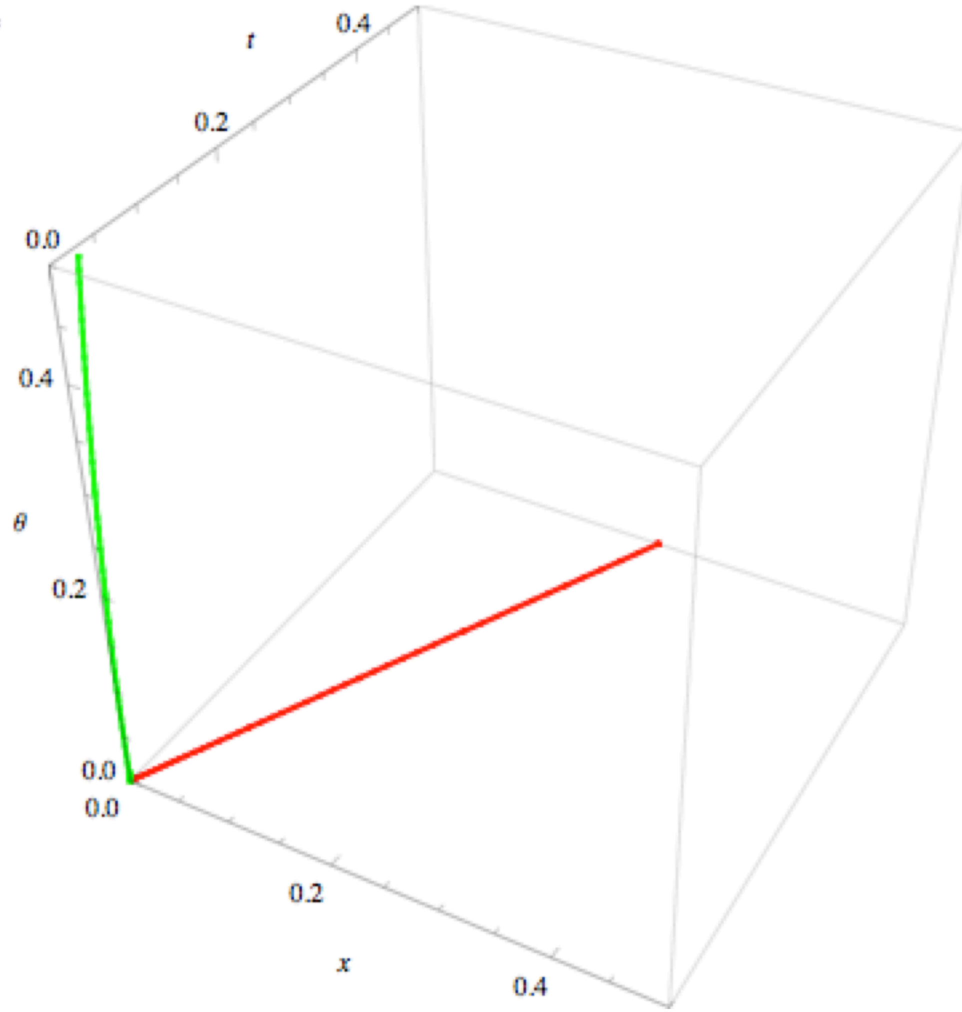

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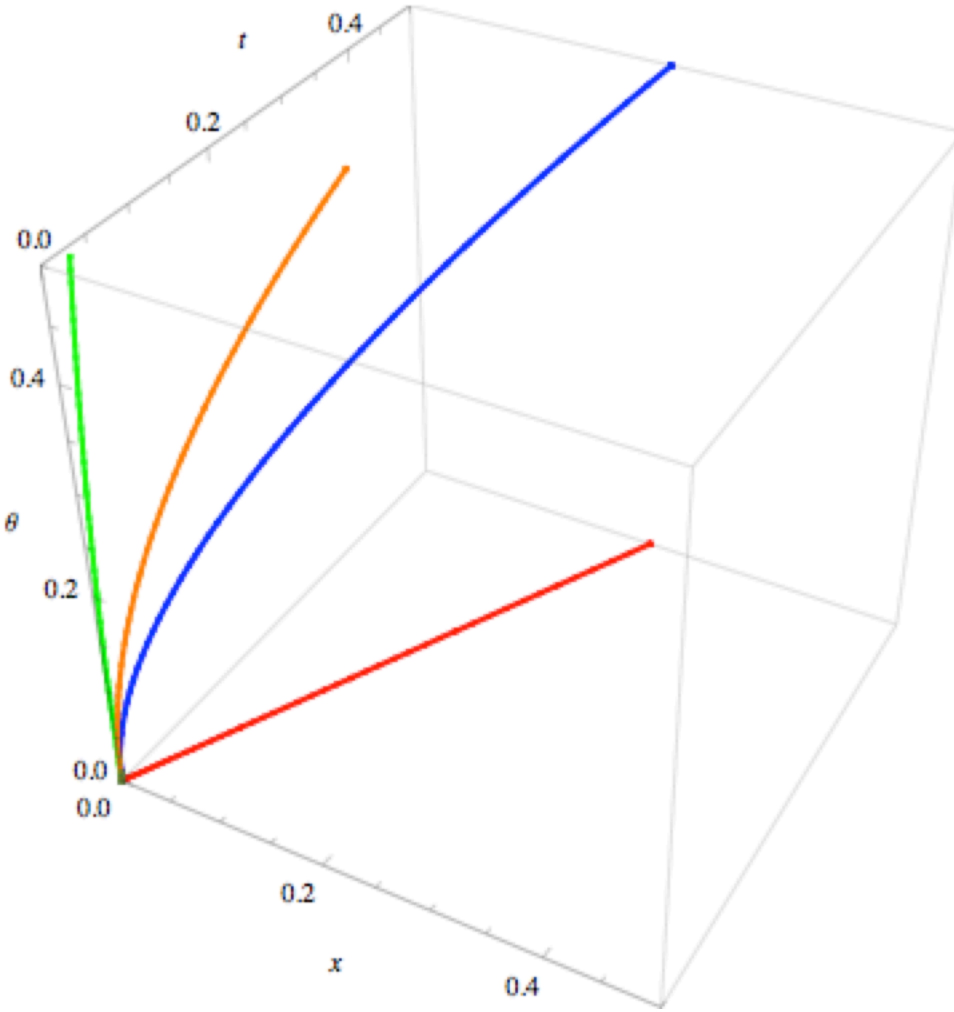

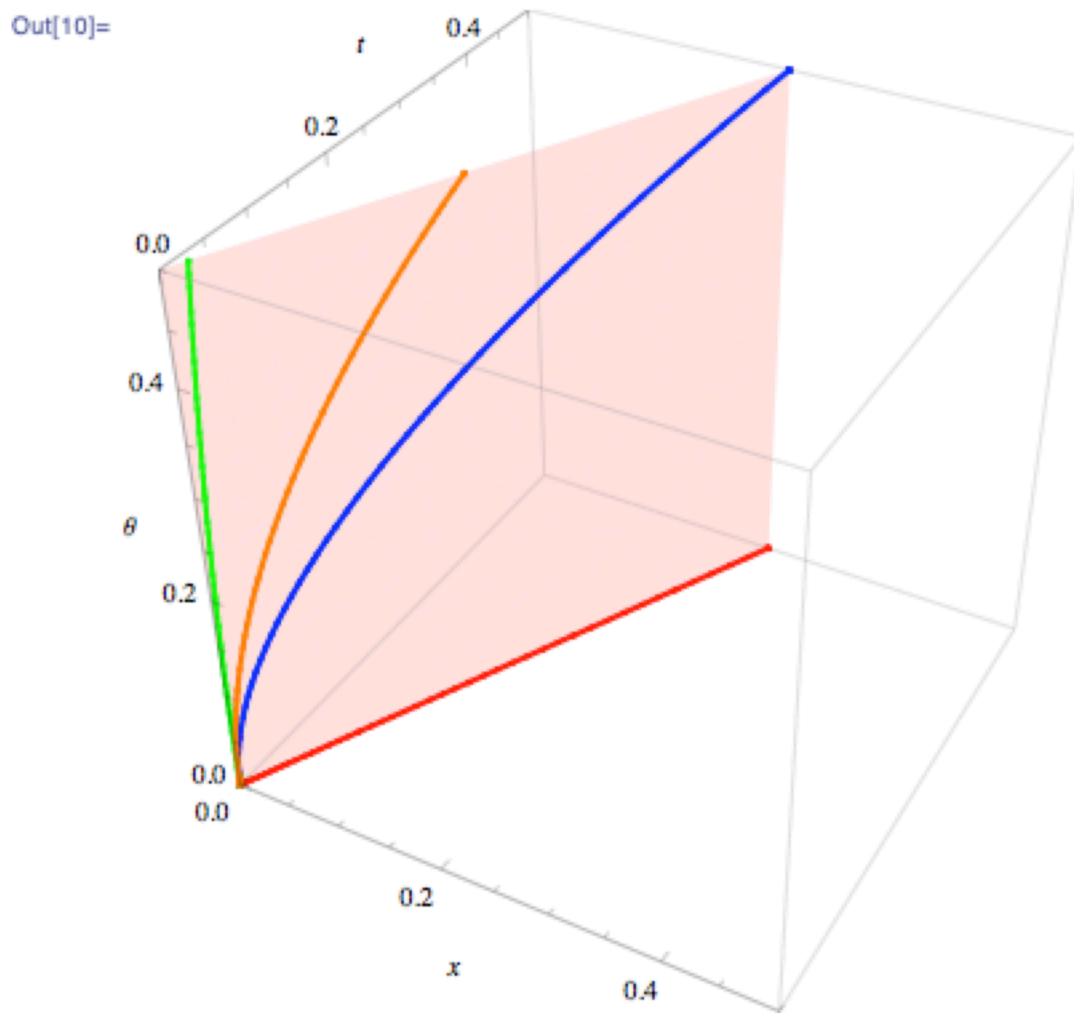
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The conjugate momentum is

$$p^\mu = mc\frac{\dot{x}^\mu}{(-\dot{x}^2)^{1/2}}$$

But the four momentum components are not independent: $p^2 + m^2c^2 \equiv 0$

Thus it is possible to solve for only three of the velocities in terms of the momentum

$$\dot{x}^a = \frac{\dot{x}^0 p^a}{(m^2 + |\vec{p}|^2)^{1/2}}$$

The resulting Hamiltonian is

$$H = p_a\dot{x}^a + p_0\dot{x}^0 - L = \dot{x}^0\left(p_0 + \left(m^2c^2 + |\vec{p}|^2\right)^{1/2}\right) = 0$$

Example 2: Free electromagnetic field

- Free electromagnetic field example. A_0 is the electrostatic potential and A_1, A_2, and A_3 the vector potential.

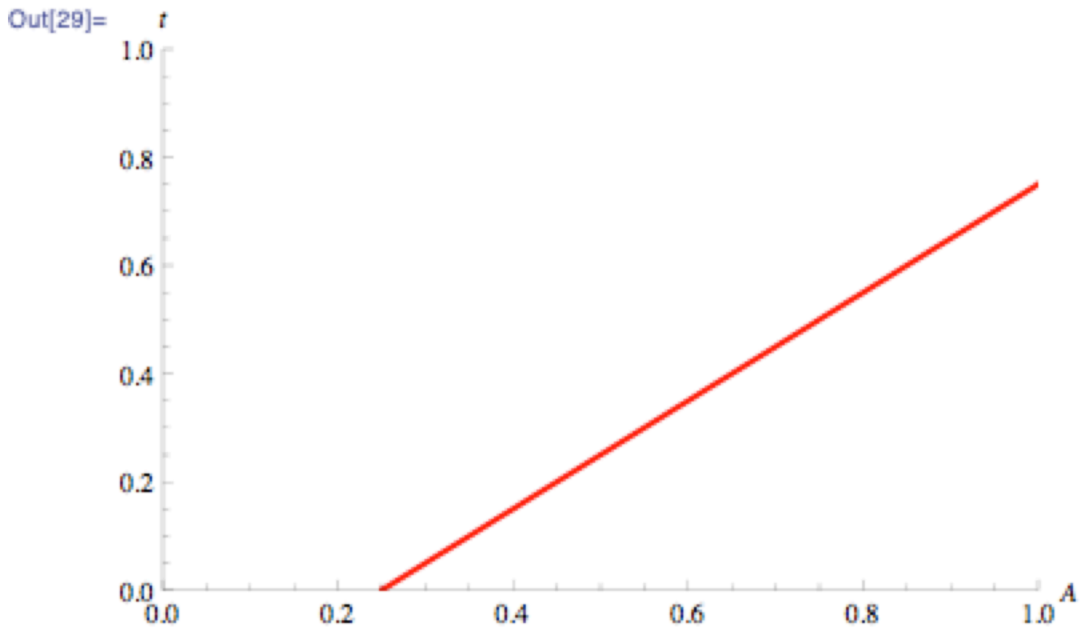
- The second order dynamical equations do not change their form under the redefinitions of the fields, where λ is an <u>arbitrary</u> function

$$A'_0 = A_0 + \frac{\partial \lambda}{\partial t} \qquad A'_a = A_a + \frac{\partial \lambda}{\partial x^a}$$

- Thus the dynamics is not deterministic. The same initial values

$$A_\mu(t_1), \frac{\partial A_\mu(t_1)}{\partial t}$$

can lead to different time evolution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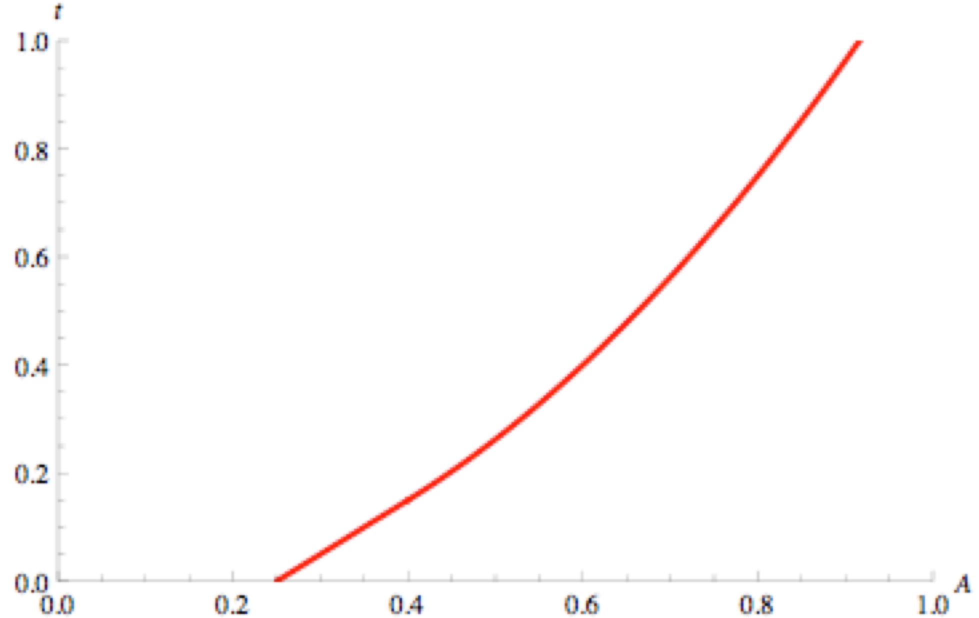

- But note that the physically measureable electric and magnetic fields do not change under a gauge transformation. Therefore their evolution in time is deterministic

$$E'^a = -\frac{\partial A'_0}{\partial x^a} + \frac{\partial A'^a}{\partial t} = -\frac{\partial A_0}{\partial x^a} + \frac{\partial A^a}{\partial t} = E^a$$

$$B'_1 = \frac{\partial A'_3}{\partial x^2} - \frac{\partial A'_2}{\partial x^3} = \frac{\partial A_3}{\partial x^2} - \frac{\partial A_2}{\partial x^3} = B_1$$

IV. Obstacles to canonical quantization of electrodynamics

Momentum conjugate to A_0 is zero

$$p^a = \frac{\partial L}{\partial \dot{A}_a} = -\frac{\partial A_0}{\partial x^a} + \frac{\partial A^a}{\partial t} = E^a \qquad p^0 = \frac{\partial L}{\partial \dot{A}_a} = 0$$

Note also that according to Gauss's Law

$$\frac{\partial E^a}{\partial x^a} = \frac{\partial p^a}{\partial x^a} = 0$$

But this is inconsistent with the quantum commutator

$$\left[A_a(x,y,z), p^b(x',y',z')\right] = i\hbar\delta_a^b \delta(x-x')\delta(y-y')\delta(z-z')$$

V. Heisenberg/Pauli and Fock approaches to quantum electrodynamics

Proposals for dealing with vanishing momentum:

Heisenberg/Pauli formalism (*Annalen der Physik*, 1929-1930)

- Add non-gauge invariant term to Lagrangian (destroys local symmetry)
- Or set $A_0 = constant$ (destroys manifest Lorentz invariance)

Pauli: “Ich warne Neugierige”

Fermi formalism (*Rend. Lincei*, 1929-1930)

- Add gauge fixing term to the Lagrangian

V - Leon Rosenfeld and his pioneering work

- Born Belgium 1904
- Doctorate Liege 1926
- Research in Paris, Göttingen, Zurich 1926-1930
- Taught theoretical physics at Liege, Utrecht, Manchester, Copenhagen 1940-1974
- Collaborators and correspondents: Bohr, Pauli, de Broglie, Dirac, Heisenberg, Infeld, Klein …
- Died October 1974

Heisenberg and Rosenfeld

Rosenfeld (right, standing)
at 1933 Solvay Meeting

VI. Rosenfeld's Constrained Hamiltonian Dynamics Formalism

Rosenfeld's debt to Pauli: "*As I was investigating these relations in the especially instructive example of gravitation theory, Professor Pauli helpfully indicated to me the principles of a simpler and more natural manner of applying the Hamiltonian procedure in the presence of identities*" (My translation from Rosenfeld's 1930 paper)

Rosenfeld focused on the construction of a phase space function that would generate gauge symmetry transformations

He was able to show using the presence of time derivatives of arbitrary functions in this generator that there will generally be additional constraints beyond those that follow from the definitions of the momenta

Rosenfeld's formal constraint analysis in "On the quantization of wave fields", Annalen der Physik 1930

Application to quantum electrodynamics addressed in "La théorie quantique des champs", Annales de l'Institut Henri Poincaré, 1932

Zur Quantelung der Wellenfelder
Von L. Rosenfeld

Einleitung

Wesentliche Fortschritte in der Formulierung der allgemeinen Quantengesetze der elektromagnetischen und materiellen Wellenfelder haben neuerdings Heisenberg und Pauli[1]) erzielt, indem sie die von Dirac erfundene „Methode der nochmaligen Quantelung" systematisch entwickelten. Neben gewissen sachlichen Schwierigkeiten, die viel tiefer liegen, trat dabei eine eigentümliche Schwierigkeit formaler Natur auf: der zum skalaren Potential kanonisch konjugierte Impuls verschwindet identisch, so daß die Aufstellung der Hamiltonschen Funktion und der Vertauschungsrelationen nicht ohne weiteres gelingt. Zur Beseitigung dieser Schwierigkeit sind bisher drei Methoden vorgeschlagen worden, die zwar ihren Zweck erfüllen, aber doch schwerlich als befriedigend betrachtet werden können.

1. Die erste Heisenberg-Paulische Methode ist ein rein analytischer Kunstgriff.[2]) Man fügt zur Lagrangefunktion gewisse Zusatzglieder hinzu, die mit einem kleinen Parameter ε multipliziert sind und bewirken, daß der obenerwähnte Impuls nicht mehr verschwindet. In den Schlußresultaten muß man dann zum Limes $\varepsilon = 0$ übergehen. Die ε-Glieder führen aber zu unphysikalischen Rechenkomplikationen[3]) und zerstören die charakteristische Invarianz der Lagrangefunktion gegenüber der Eichinvarianzgruppe.

2. Die zweite Heisenberg-Paulische Methode[4]) benutzt hingegen wesentlich diese Invarianz. Dem skalaren Potential

1) W. Heisenberg u. W. Pauli, Ztschr. f. Phys. **56**. S. 1. 1929; ebenda **59**. S. 168. 1930. Im folgenden mit H. P. I bzw. II zitiert.

2) H. P. I, S. 24—26, 30ff.

3) Vgl. L. Rosenfeld, Ztschr. f. Phys. **58**. S. 540. 1929.

4) H. P. II.

Rosenfeld's formal constraint analysis in "On the quantization of wave fields", Annalen der Physik 1930

- Local symmetries always lead to
  - non-unique evolution in time
  - constraining relations among variables and associated momenta
  - Hamiltonian (from which equations of motion are determined) constructed using the constraints
  - vanishing of Hamiltonian if, as in general relativity, the equations of motion take the same form for arbitrary choices of the time coordinate
- Rosenfeld was first to consider how to implement local symmetry-induced transformations on Hamiltonian variables
- Rosenfeld's dynamical model - gravitation with a charged spinorial

Dirac field source

Origins of the model

- Weyl/Fock coupling of Dirac field with gravity - 1929
- Tetrads and Weyl's reinterpretation of gauge symmetry
  See analyses by Scholz (physics/0409158) and Straumann (hep-ph/0509116)

- The Rosenfeld method is strictly valid for conventional local gauge theories, including all special relativisitic gauge theories.

- In particular, Rosenfeld was able to prove that the special choices made by Pauli, Heisenberg, Fermi - and later also by Dirac - were mathematically justified. The formalism demonstrated that gauge choices did not eliminate any distinct physically realizeable solutions.
- Rosenfeld himself ceased to use the formalism in his later work since the earlier, no longer questionable methods were mathematically much simpler.
- The expression for the generator of general coordinate transformations and the derivation of new constraints is not correct. Rosenfeld failed to notice that there are some functions of position and velocity variables that have no counterpart in phase space. This occurs, for example, with the parameterized free particle. Notice when there does exist such a correspondence, for example,

$$\phi(x, p(x, \dot{x})) =: \bar{\phi}(x, \dot{x})$$

then $$\frac{\partial \bar{\phi}}{\partial \dot{x}^\mu} = \frac{\partial \phi}{\partial p_\nu} \frac{\partial p_\nu}{\partial \dot{x}^\mu}$$

But since $p^\nu \frac{\partial p_\nu}{\partial \dot{x}^\mu} = 0$ it follows that $p^\mu \frac{\partial \bar{\phi}}{\partial \dot{x}^\mu} = 0$

VI. The impact of Rosenfeld's work

- Pauli and Heisenberg seem not to have fully appreciated Rosenfeld's work
- In a letter to O. Klein in 1955 Pauli writes "I would like to bring to your attention the work by Rosenfeld in 1930. He was known here at the time as the `man who quantised the Vierbein' (sounds like the title of a Grimm's fairy tale doesn't it?) … Ich still remember that I was not happy with every aspect of his work since he had to introduce certain additional assumptions that no one was satisfied with."
- Pauli worked out a completely gauge invariant formalism, using only electric and magnetic fields (that he "shared" with Dirac in a private communication in 1932)

- Rosenfeld brought his work to the attention of Paul Dirac in May, 1932. Dirac responded with specific questions concerning his arbitrary functions

> I have been studying your papers, but have had some trouble in understanding the significance of your λ's. What exactly is meant by the statement that they are arbitrary? On page 143 of your Annalen paper, the first of equations (III), when worked out, gives
>
> $$\frac{\partial Q_{\bar{\nu}}}{\partial t} = \frac{\partial Q_{\bar{\nu}}}{\partial t} - \frac{\partial Q_4}{\partial x_{\bar{\nu}}}$$
>
> or $$\frac{\partial Q_4}{\partial x_{\bar{\nu}}} = 0$$

- Peter Bergmann was unaware of Rosenfeld's work when he published his first two papers 1949 dealing with the Hamiltonian formulation of generally covariant theories.

- Dirac did not refer to Rosenfeld in his own work on generalized Hamiltonian dynamics, beginning also in 1949, until Bergmann and collaborators began to cite him.

- The procedure is now known variously as the Dirac, or Dirac/Bergmann, procedure.

- Constrained Hamiltonian dynamics only became a member of the standard toolkit for theoretical physics following Gerard t'Hooft 1971 proof, employing gauge symmetry, of the renormalizability of non-Abelian gauge theories.