Léon Rosenfeld's pioneering steps toward a quantum theory of gravity

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Plan of Talk

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3. Léon Rosenfeld background
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   - Gauge generator and higher-order constraints
   - Problems with projectability and contemporary resolution
5. Rosenfeld's unified model
6. Twenty years of quantum gravity: 1930-1950
7. Dirac, Bergmann, and Rosenfeld
8. Conclusions

1. An abbreviated history of GR and quantum mechanics

- 1915 - Einstein's general theory of relativity
- 1916 - Einstein's first observation of need for quantization of gravity: "…it appears that the quantum theory will need to modify not only Maxwellian electrodynamics, but also the new gravitation theory."
- 1923 - de Broglie matter waves
- 1925 - Heisenberg matrix mechanics
- 1926 - Schroedinger wave mechanics; Born, Heisenberg and Jordan (Dreimännerarbeit) first field quantization
- 1927 - Dirac quantization of the electromagnetic field
- 1928 - relativistic Dirac equation; Born and Jordan second quantization of the free electromagnetic field
- 1929 - Heisenberg and Pauli second quantization of relativistic Dirac matter field in interaction with electromagnetic field

Whimsical Aside

Peter Bergmann ca. 1925
(Although he was a child prodigy - I guess we can assume that he was not engaged with the old quantum theory.) (Photo courtesy of Ernest Bergmann)

Max Bergmann and son Peter ca. 1921. Max was Director of the Kaiser Wilhelm Institute for Leather Research from 1922 to 1933. In the 1920's and 30's he was the world's leading pioneering researcher in protein chemistry. (Photo courtesy of Ernest Bergmann)

2. The Pauli-Heisenberg connection

Pre-history of quantum constrained field dynamics - 1929

(See D.S. arXiv:0904.3993 - to appear in Studies in Hist. Phil. Mod. Phys.)

The vanishing momentum problem in quantum electrodynamics

The original choice for the many-body Lagrangian was

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - eA_\mu\bar{\psi}\gamma^\mu\psi + i\hbar c\bar{\psi}\gamma^\mu\psi_{,\mu} - mc^2\bar{\psi}\psi$$

The corresponding momentum conjugate to A_μ is

$$p^\mu = \frac{\partial\mathcal{L}}{\partial\dot{A}_\mu} = F^{0\mu} \text{ so } p^0 \equiv 0$$

But a vanishing p^0 is inconsistent with the commutation relations

$$\left[A_0(x), p^0(x')\right] = i\hbar\delta^3(x - x')$$

Heisenberg's *Kunstgriff* - Pauli and Heisenberg I (1929)

Destroy invariance under gauge transformations $\delta A_\mu = \xi_{,\mu}$ and $\delta\psi = \frac{ie}{\hbar c}\psi\xi$
by adding a gauge breaking term $\frac{\epsilon}{2}(A^\mu_{,\mu})^2$ to the Lagrangian:

$$\mathcal{L}' = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - eA_\mu\bar{\psi}\gamma^\mu\psi + i\hbar c\bar{\psi}\gamma^\mu\psi_{,\mu} - mc^2\bar{\psi}\psi + \frac{\epsilon}{2}\left(A^\mu_{,\mu}\right)^2$$

Then take the limit $\epsilon \to 0$ at completion of calculations

Heisenberg and Pauli's self-assessments

Also in those papers which Pauli and I wrote on the quantization of fields we saw quite soon that after all it didn't look too well. It is true that for the free light quanta everything could be made to fit, but as soon as interaction came in it didn't look right (Heisenberg, Archive for the History of Quantum Mechanics (AHQM) 2/28/63, p 22)

Here, in electrodynamics, it didn't become simple… For instance, you had to introduce this supplementary condition and you had to make some kind of limiting process -- first introducing an epsilon and at the end you put epsilon to zero. You know, that kind of stuff didn't look right. (Heisenberg, AHQM 2/28/63, p 23)

Already there it was a bit artificial to do the Lorenz condition without introducing the indefinite metric. Well, finally Pauli and I succeeded in replacing it by some symmetry argument, but again it was a bit funny. You could say that the fourth Maxwell equation is not a rigorous operator equation, it's only a supplementary condition to the --, Well, you know. It came into the region of the "Ausrede." (Heisenberg, AHQM 7/28/63, p 7)

3. Leon Rosenfeld background

Born Belgium 1904
Doctorate Liege 1926
Research in Paris under direction of de Broglie, Brillouin, and Langevin 1926-27
Research in Göttingen under direction of Born 1927-28
Research in Zürich under Pauli 1929-30
Beginning in 1930 several months in Copenhagen in collaboration with Bohr
Taught theoretical physics at Liege 1930-37, Utrecht 1940-47, Manchester 1947-58
NORDITA, Copenhagen 1958-197
Collaborators and correspondents: Bohr, Pauli, de Broglie, Dirac, Heisenberg, Infeld, Klein
…Died October 1974

Rosenfeld and Pauli

I came to Zürich before the summer semester… I came from Göttingen where I was still at the time. I had already corresponded with Bohr, asking him whether I could come to Copenhagen… and so I wrote to Pauli then to ask him if he would take me up. He was very friendly and he said: "With pleasure, because we have just completed a scheme of quantum electrodynamics with Heisenberg; 'dass ist ein Gebiet, dass noch nicht abgebrochen ist.'" So he was eager to have people brush up the details and explore the consequences and that is what I did at Zürich actually (AHQM 7/19/63, p 5)

… I got provoked by Pauli to tackle this problem of the quantization of gravitation and the gravitation effects of light quanta, which at that time were more interesting. When I explained to Pauli what I wanted to work out, I think it was the Kerr effect or some optical effect, he said "Well, you may do that, and I am glad beforehand for any result you may find." That was a way of saying that this was a problem that was not instructive, that any result might come out, whereas at that time, the calculation of the self energy of the light quantum arising from its gravitational field was done with a very definite purpose. (AHQM 7/19/63, p 8)

…Then Pauli told me that he was not at all pleased with longitudinal waves, so he wanted to have them treated another way, which I did, but that was not more enlightening, far from it. (AHQM 7/19/63, p 9)

… There was this point in their proof in which the invariants of the Hamiltonian seemed to depend on a special structure of the Hamiltonian, and that looked suspicious… "Yes, I understand that [said Pauli], but we have not been able to find a mistake in our calculation and we do not understand what this means; we suspect that it must be wrong, but we don't know." Then the thing came to a crisis through the fact that I tried to make a more general formulation of field quantization … It was a purely abstract scheme which worked in a completely general way with only this complication of accessary conditions, but at any rate, not due to any special structure but only to the existence of invariance with respect to a group. So at that stage I was convinced that there must be a mistake in the original paper… (AHQM 7/19/63, p 5)

4. Rosenfeld's formal constraint analysis in "On the quantization of wave fields", Annalen der Physik, 113 - 152 (1930)

(Annotated translation soon available as MPIWG Berlin preprint)

Application to quantum electrodynamics addressed in "La théorie quantique des champs", Annales de l'Institut Henri Poincaré, 25 - 91 (1932)

Zur Quantelung der Wellenfelder

Von L. Rosenfeld

Einleitung

Wesentliche Fortschritte in der Formulierung der allgemeinen Quantengesetze der elektromagnetischen und materiellen Wellenfelder haben neuerdings Heisenberg und Pauli[1]) erzielt, indem sie die von Dirac erfundene „Methode der nochmaligen Quantelung" systematisch entwickelten. Neben gewissen sachlichen Schwierigkeiten, die viel tiefer liegen, trat dabei eine eigentümliche Schwierigkeit formaler Natur auf: der zum skalaren Potential kanonisch konjugierte Impuls verschwindet identisch, so daß die Aufstellung der Hamiltonschen Funktion und der Vertauschungsrelationen nicht ohne weiteres gelingt. Zur Beseitigung dieser Schwierigkeit sind bisher drei Methoden vorgeschlagen worden, die zwar ihren Zweck erfüllen, aber doch schwerlich als befriedigend betrachtet werden können.

1. Die erste Heisenberg-Paulische Methode ist ein rein analytischer Kunstgriff.[2]) Man fügt zur Lagrangefunktion gewisse Zusatzglieder hinzu, die mit einem kleinen Parameter ε multipliziert sind und bewirken, daß der obenerwähnte Impuls nicht mehr verschwindet. In den Schlußresultaten muß man dann zum Limes $\varepsilon = 0$ übergehen. Die ε-Glieder führen aber zu unphysikalischen Rechenkomplikationen[3]) und zerstören die charakteristische Invarianz der Lagrangefunktion gegenüber der Eichinvarianzgruppe.

2. Die zweite Heisenberg-Paulische Methode[4]) benutzt hingegen wesentlich diese Invarianz. Dem skalaren Potential

1) W. Heisenberg u. W. Pauli, Ztschr. f. Phys. **56**. S. 1. 1929; ebenda **59**, S. 168. 1930. Im folgenden mit H. P. I bzw. II zitiert.

2) H. P. I, S. 24—26, 30ff.

3) Vgl. L. Rosenfeld, Ztschr. f. Phys. **58**. S. 540. 1929.

4) H. P. II.

Rosenfeld's debt to Pauli:

As I was investigating these relations in the especially instructive example of gravitation theory, Professor Pauli helpfully indicated to me the principles of a simpler and more natural manner of applying the Hamiltionian procedure in the presence of identities. This procedure is not subject to the disadvantages of the earlier methods.

Rosenfeld considers coordinate transformations of the form

$$\delta x^\nu = a_r^{\nu,0}(x)\epsilon^r(x) + a_r^{\nu,\sigma}(x)\frac{\partial \epsilon^r}{\partial x^\sigma} + a_r^{\nu,\sigma\cdots\tau}(x)\frac{\partial^k \epsilon^r}{\partial x^\sigma \cdots \partial x^\tau}$$

(We will be concerned exclusively (until later) with special case $\delta x^\nu = \epsilon^\nu(x)$ but will have additional internal gauge freedom $\epsilon^r, \ r > 3$)

Gauge symmetry transformations of the form
(where the $\epsilon^r(x)$ are arbitrary)

$$\delta Q_\alpha = c_{\alpha r}^0(x,Q)\epsilon^r(x) + c_{\alpha r}^\sigma(x,Q)\frac{\partial \epsilon^r}{\partial x^\sigma}$$

(Rosenfeld actually considered more general variations depending on higher derivatives of the descriptors.)

Conventional gravity example:

Write metric in terms of lapse N and shift N^a

$$g_{\mu\nu} = \begin{pmatrix} -N^2 + N^c N^d g_{cd} & g_{ac} N^c \\ g_{bd} N^d & g_{ab} \end{pmatrix}$$

Under the infinitesmal coordinate transformation $\quad x'^\mu = x^\mu - \epsilon^\mu(x)$

$$\delta g_{\mu\nu} = g_{\mu\alpha}\epsilon^\alpha_{,\nu} + g_{\alpha\nu}\epsilon^\alpha_{,\mu}$$

$$\delta N = N\dot{\epsilon}^0 - N N^a \epsilon^0_{,a} \qquad \delta N^a = N^a \dot{\epsilon}^0 - (N^2 e^{ab} + N^a N^b)\epsilon^0_{,b} + \dot{\epsilon}^a - N^b \epsilon^a_{,b}$$

Rosenfeld considers Lagrangians quadratic in field derivatives

$$\mathcal{L} = \frac{1}{2}\left(Q_{\alpha,\nu}\mathcal{A}^{\alpha\nu,\beta\mu}(Q)Q_{\beta,\mu} + Q_{\alpha,\nu}\mathcal{B}^{\alpha\nu}(Q) + \mathcal{B}^{\alpha\nu}(Q)Q_{\alpha,\nu} + \mathcal{C}(Q)\right)$$

He then supposes that the variations δQ_α are Noether symmetry transformations, so that the Lagrangian transforms as a scalar density:

$$\delta\mathcal{L} + \mathcal{L}\frac{\partial \delta x^\mu}{\partial x^\mu} \equiv 0$$

(E. Noether, 1918 - Rosenfeld also treats the more general case in which a total divergence arises.)

Example: $\mathcal{L} = \sqrt{-g}R$, where

$$R = {}^{(3)}R + {}^{(3)}g^{ac\,(3)}g^{bd}K_{ab}K_{cd} - {}^{(3)}g^{ab\,(3)}g^{cd}K_{ab}K_{cd} + (n^\mu n^\nu_{|\nu})_{|\mu} - (n^\nu n^\mu_{|\mu})_{|\nu}$$

with extrinsic curvature $K_{ab} = \frac{1}{2N}(\dot{g}_{ab} - D_a N_b - D_b N_a)$

and normal to the fixed time hypersurface $n^\mu = \left(N^{-1}, -N^{-1}N^a\right)$

(In following I will subtract total divergence: $\mathcal{L}_G = N\sqrt{{}^{(3)}g}\left({}^{(3)}R + K_{ab}K^{ab} - K^2\right)$)

4a. Construction of Hamiltonian

The key observation: in the variation of the Lagrangian under symmetry transformations the coefficients of the highest time derivatives of the arbitrary functions ε vanish identically.

First note that the canonical momentum is: $\mathcal{P}^\alpha = \dfrac{\partial \mathcal{L}}{\partial \dot{Q}_\alpha} = \mathcal{A}^{\alpha\nu,\mu 0} Q_{\alpha,\nu}$

I -Then writing the relevant term in the variation we get primary constraints:

$$\delta \mathcal{L} = \mathcal{P}^\mu c^0_{\mu r} \ddot{\epsilon}^r + \cdots \Rightarrow \mathcal{P}^\mu c^0_{\mu r} \equiv 0$$

II - Note also that the primary constraints give us null vectors of the Legendre matrix :

$$\mathcal{A}^{\alpha 0,\mu 0} c^0_{\mu r} \equiv 0$$

III - Since $p^\alpha = \mathcal{A}^{\alpha 0,\mu 0} \dot{Q}_\mu + \cdots$ velocities are not uniquely fixed in terms of the momenta:

$$\dot{Q}_\mu = \frac{\partial\, {}^0\mathcal{H}}{\partial \mathcal{P}^\mu} + \lambda^r c^0_{\mu r} = \frac{\partial \left({}^0\mathcal{H} + \lambda^r \mathcal{P}^\nu c^0_{\nu r} \right)}{\partial \mathcal{P}^\mu} = \lambda^r \frac{\partial \mathcal{H}}{\partial \mathcal{P}^\mu}$$

λ^r are arbitrary functions

$\mathcal{H}_0$ is obtained by finding a particular solution ${}^0\dot{Q}_\mu$ of the defining relation

$$\mathcal{P}^\alpha = \mathcal{A}^{\alpha 0,\mu 0}\,{}^0\dot{Q}_\mu + \mathcal{A}^{\alpha 0,\mu a}\,{}^0Q_{\mu,a}$$

Then compute the canonical Hamiltonian ${}^0\mathcal{H} = \mathcal{P}^\alpha\,{}^0\dot{Q}_\alpha - \mathcal{L}(Q, {}^0\dot{Q})$

Example - Observation I: Primary constraints $p_\mu = \dfrac{\partial \mathcal{L}_G}{\partial \dot{N}^\mu} = 0$

Observation II: Null vectors $\dfrac{\partial}{\partial \dot{N}^\mu}$

Observation III: Hamiltonian $\mathcal{H} = N^\mu \mathcal{H}_\mu + \lambda^\mu p_\mu$

$\mathcal{H}_a$ is the usual scalar constraint, but the vector constraint differs from the conventional by a total spatial divergence: $\mathcal{H}_a = 2p^{ab} D_a N_b$

Note that equations of motion for lapse and shift yield $\dot{N}^\mu = \lambda$

4b. The canonical generator of active gauge transformations

Active variations are variations in functional form (Lie derivative along vector field $\epsilon^\mu = -\delta x^\mu$).

$$\bar{\delta} Q_\alpha(x) = Q'_\alpha(x) - Q_\alpha(x) \quad \bar{\delta}\mathcal{P}^\alpha(x) = \mathcal{P}'^\alpha(x) - \mathcal{P}^\alpha(x)$$

(This notation was apparently borrowed by Bergmann from Noether (1918))

Rosenfeld proved that the following integral generates the correct active gauge variations of Q and $\mathcal{P}$.

$$\overline{\mathcal{M}} = \int d^3x \, \left(\mathcal{P}^\alpha \delta Q_\alpha - \mathcal{H}\delta x^0 - \mathcal{P}^a Q_{,a}\delta x^a\right)$$

Rosenfeld then showed that this generator is time-independent: $\frac{d\overline{\mathcal{M}}}{dt} = 0$

Consequently, the coefficients of the time derivatives of each order must vanish. Rosenfeld then proved that this generator could always be written as the sum of time derivatives of the primary constraints multiplying time derivatives of the arbitrary function ϵ^μ.

$$\overline{\mathcal{M}} = \int d^3x \left(\frac{d\epsilon^r}{dt} p^\mu c^0_{\mu r} - \epsilon^r \frac{d}{dt}\left(p^\mu c^0_{\mu r}\right)\right)$$

Thus Rosenfeld showed that the preservation in time of primary constraints leads to secondary (and tertiary …) constraints. This result has always been attributed to Anderson and Bergmann. The terminology is due to them and was employed later by Dirac. (J. L. Anderson and P. G. Bergmann, PR 83, 1018 (1951)

Example:

$$\mathcal{M} = \frac{1}{2}p^{ab}\delta g_{ab} + p_\mu \delta N^\mu - \mathcal{H}\delta x^0 - \frac{1}{2}p^{ab}g_{ab,c}\delta x^c - p_\mu N^\mu_{,c}\delta x^c$$

$$\begin{aligned}
&= p^{ab}\left(g_{ca}\epsilon^c_{,b} + N^c g_{ca}\epsilon^0_{,b}\right) + p_0\left(N\dot{\epsilon}^0 - NN^a\epsilon^0_{,a}\right) + p_a\left(N^a\dot{\epsilon}^0 - (N^2 e^{ab} + N^a N^b)\epsilon^0_{,b} + \dot{\epsilon}^a - N^b\epsilon^a_{,b}\right) \\
&+ \left(N\mathcal{H}_0 + N^a\mathcal{H}_a + \dot{N}^\mu p_\mu\right)\epsilon^0 + \frac{1}{2}p^{ab}g_{ab,c}\epsilon^c + p_0 N_{,c}\epsilon^c + p_a N^a_{,c}\epsilon^c
\end{aligned}$$

4c. Problems with the canonical gauge generator for generally covariant systems

Rosenfeld did not take into account the requirement that the generator must be projectable under the Legendre transformation from configuration/velocity space to phase space.

Bergmann and Brunings were apparently the first to note this requirement in print: (P. G. Bergmann and J. H. M. Brunings, Rev Mod Phys 21, 480 (1949). Lee and Wald were the first to begin systematic exploration of this condition. (J. Lee and R. M. Wald, JMP 31, 725 (1990))

Example: Note that the $\dot{N}^\mu$ terms are not projectable.

Projectability is attained through a compulsory dependence of the infinitesimal coordinate transformation descriptors on the lapse and shift. (J. M. Pons, D. S., L. S. Shepley, PRD 55, 658 (1997)) $\epsilon^\mu = \delta^\mu_a \xi^a + n^\mu \xi^0$

Substitution into Rosenfeld's generator yields the 1997 result:

$\mathcal{M} = p_\mu \dot{\xi}^\mu + (\mathcal{H}_\mu + N^\rho C^\nu_{\mu\rho} p_\nu)\xi^\mu$ where $\{\mathcal{H}_\mu, \mathcal{H}_\nu\} = C^\alpha_{\mu\nu}\mathcal{H}_\alpha$

Further modifications in Rosenfeld's generator are required if there exist additional gauge symmetries beyond diffeomorphism symmetry. It turns out that pure (lapse and shift dependence) diffeomorphisms cannot be realized as canonical transformations. An internal gauge transformation must be added. This has been known in various guises since the 1970's. A group-theoretical explanation was given in 1983 for Einstein-Yang-Mills theory. (K. Sundermeyer, D.S., PRD 27, 757 (1983)). A projectivity analysis followed in 2000. (J. M. Pons, D. S., L. S. Shepley (2000))

The Rosenfeld generator is correct provided the appropriate gauge transformation is added to the variation under diffeomorphisms.

Einstein-Yang-Mills example:

$$\begin{aligned}\delta A^i_0 &= A^i_\nu (n^\nu \xi)_{,0} \dot{\xi} \\ &= A^i_0 (-N^{-2} \dot{N} \xi + N^{-1} + A^i_b (-N^{-2} \dot{N} N^b \xi + N^{-1} \dot{N}^b \xi + N^{-1} N^b \dot{\xi})\end{aligned}$$

The $\dot{N}^\mu$ terms are not permitted. In addition the Hamiltonian contains $\dot{A}^i_0$ that must also be removed. This feat is accomplished (uniquely) by supplementing the diffeomorphism variation with a gauge transformation with descriptor $\Lambda^i = A^i_\mu n^\mu \xi^0$

The corresponding Yang-Mills gauge transformation is $\delta_G A^i_0 = -\Lambda^i_{,0} - C^i_{jk} \Lambda^j A^k_0$

With this modification the Rosenfeld generator agrees with the 2000 result.

5. Rosenfeld's unified Lagrangian

Quadratic tetrad gravitational contribution

$$\mathcal{L}_G = (-g)^{\frac{1}{2}} E^\mu_I E^\nu_J \left(\omega_\mu{}^I{}_L \omega_\nu{}^{LJ} - \omega_\nu{}^I{}_L \omega_\mu{}^{LJ}\right)$$

where $\omega_\nu{}^{LJ}$ *are the Ricci rotation coefficients and* E^μ_I *are the tetrad fields with Minkowski index I.*

Minimal coupling to a Dirac electron field

$$\mathcal{L}_M = i\hbar c(-g)^{1/2} \overline{\psi} E^\mu_L \Gamma^L \left(\frac{\partial}{\partial x^\mu} + \Omega_\mu - i\frac{e}{\hbar c}\phi_\mu\right)\psi + mc^2\overline{\psi}\psi(-g)^{1/2}$$

where Ω_μ *is the spinor connection, first obtained independently by Weyl and Fock* *(*H. Weyl, Zeitschrift für Physik 56, 330 (1929) and V. Fock, ZP 57, 261 (1929))

$$\Omega_\mu = \frac{1}{4}\gamma^I\gamma^J\omega_{\mu IJ}$$

Rosenfeld constructed the Hamiltonian and the full gauge generator for a q-number version of this model! He did not, however, display explicitly the phase space form of either the gravitational Hamiltonian or the generators.

6. Twenty years of quantum gravity: 1930 - 1950

Rosenfeld adopts Fock quantization approach in both quantum electrodynamics and quantum gravity. "On the gravitational effect of light," ZP 65, 589 (1930)

Bohr and Rosenfeld measurability analysis of the electric and magnetic fields (1933).

Landau, Bohr, Rosenfeld, and Bronstein in Kharkov, Russia in 1934 (from Gorelik, http://people.bu.edu/gorelik/cGh_Bronstein_UFN-200510_Engl.htm)

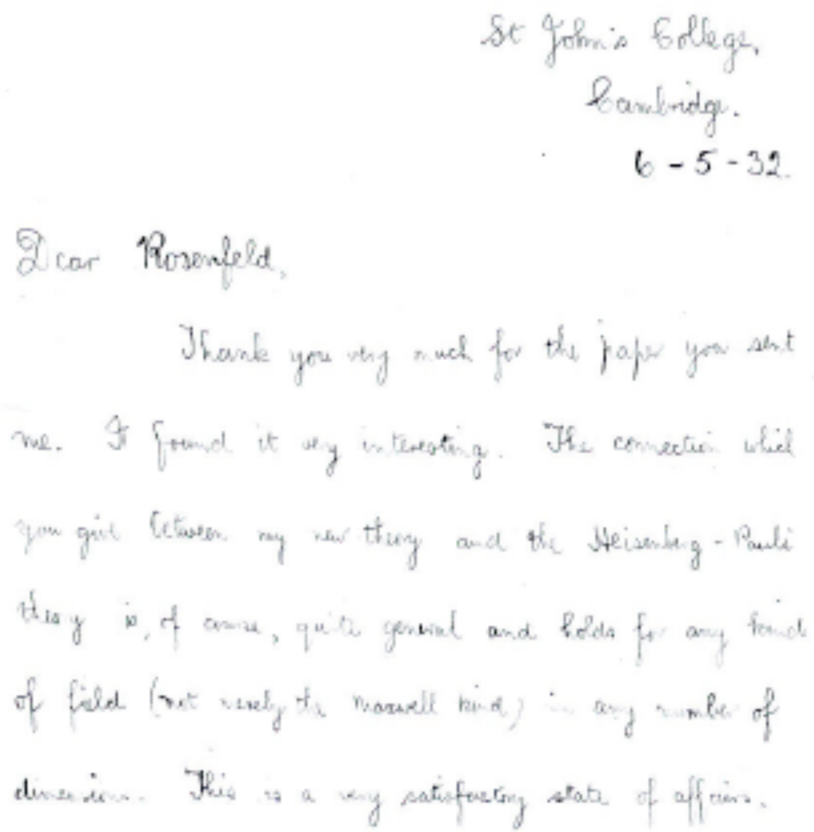
M. Bronstein, "Quantum theory of weak gravitational fields," Physikalische Zeitschrift der Sowjetunion 9, 140 (1936). Applied linear gravitational generalization of the Fock quantization procedure that had received its group theoretical justification from Rosenfeld's 1930 paper.

6a. Rosenfeld and Dirac

Dirac to Rosenfeld, 4/26/31: Many thanks for sending a copy of your paper on radiation theory, which I have read with great interest. (Niels Bohr Archive)

Rosenfeld to Dirac, 4/30/32: I enclose a note about your new theory, which is clearly not at all meant "um zu kritisieren" but "nur um zu lernen". (Churchill College Archive)

Rosenfeld publishes demonstration of equivalence of Heisenberg-Pauli and Dirac many-body theory in 1932 - submitted May 2.



St John's College,
Cambridge.
6 - 5 - 32.

Dear Rosenfeld,

Thank you very much for the paper you sent me. I found it very interesting. The connection which you give between my new theory and the Heisenberg - Pauli theory is, of course, quite general and holds for any kind of field (not merely the Maxwell kind) in any number of dimensions. This is a very satisfactory state of affairs.

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Dirac published an "improved" demonstration with Fock and Podolsky later in 1932

Rosenfeld to Dirac, (5/10/1932) … As to the doubtful sentence of Heisenberg-Pauli, which you are right in not understanding, I would suggest to you to examine the general invariance proof which I give in my paper of the "Annalen der Physik", 5, 113, 1930. (I sent you reprints of both). (Niels Bohr Archive)

I have been studying your papers, but have had some trouble in understanding the significance of your λ's. What exactly is meant by the statement that they are arbitrary? On page 143 of your Annalen paper, the first of equations (III), when worked out, gives

Dirac to Rosenfeld, (5/16/1932) … I have been studying your papers, but have had some trouble in understanding the significance of your λ's. What exactly is meant by the statement that they are arbitrary? (Niels Bohr Archive)

Rosenfeld to Dirac, (5/21/1932) … As to the λ's, they enter as arbitrary or undetermined coefficients (depending on coordinates) in the general expression of the $\dot{Q}$ *in terms of the Q's and P's. In equation (111) the hamiltonian should be the same as that of Heisenberg-Pauli (as stated there), so that the substitution of the P's in terms of the* $\dot{Q}$ *in them will lead to identities, and this implies no restriction for λ (Churchill College Archive)*

The impact of Rosenfeld's work on constrained Hamiltonian dynamics.

- Pauli to O. Klein (1/25/1955)

I would like to bring to your attention the work by Rosenfeld in 1930. He was known here at the time as the `man who quantised the Vierbein' (sounds like the title of a Grimm's fairy tale doesn't it?) See part II of his work where the Vierbein appears. Much importance was given at that time to the identities among the p's and q's (that is the canonically conjugate fields) that arise from the existance of the group of general coordinate transformations. I still remember that I was not happy with every aspect of his work since he had to introduce certain additional assumptions that no one was satisfied with.

A perhaps pertinent remark of Dirac

…Well, I think I might answer you in much the same way that I wrote that I felt it had probably been done before, but it was less trouble to me to present it as something new than to search for a reference. A good deal of my work was like that. It happened rather often that there was something which I thought had been done before, but it seemed a great nuisance to look through all the references to try to find it, and if it doesn't take much trouble to publish it, one can publish it again without claiming either that it is new or that it has been done before. (AHQM, 5/10/1963, p 15)

7b - Bergmann chronology

PHYSICAL REVIEW VOLUME 75, NUMBER 4 FEBRUARY 15, 1949

Non-Linear Field Theories

PETER G. BERGMANN
Department of Physics, Syracuse University, Syracuse, New York
(Received June 8, 1948)

This is the first paper in a program concerned with the quantization of field theories which are covariant with respect to general coordinate transformations, like the general theory of relativity. All these theories share the property that the existence and form of the equations of motion is a direct consequence of the covariant character of the equations. It is hoped that in the quantization of theories of this type some of the divergences which are ordinarily encountered in quantum field theories can be avoided. The present paper lays the classical foundation for this program: It examines the formal properties of covariant field equations, derives the form of the conservation laws, the form of the equations of motion, and the properties of the canonical momentum components which can be introduced.

REVIEWS OF MODERN PHYSICS VOLUME 21, NUMBER 3 JULY, 1949

Non-Linear Field Theories II. Canonical Equations and Quantization*

PETER G. BERGMANN AND JOHANNA H. M. BRUNINGS
Department of Physics, Syracuse University, Syracuse, New York

PHYSICAL REVIEW VOLUME 80, NUMBER 1 OCTOBER 1, 1950

The Hamiltonian of the General Theory of Relativity with Electromagnetic Field*

PETER G. BERGMANN, ROBERT PENFIELD, RALPH SCHILLER, AND HENRY ZATZKIS
Department of Physics, Syracuse University, Syracuse, New York
(Received April 24, 1950)

In this paper we have given a specific example of a Hamiltonian of a non-linear field theory, a Hamiltonian density completely free of time derivatives. In accordance with the general theory developed previously, this Hamiltonian is one of the constraints between the canonical variables and, therefore, vanishes everywhere. To obtain this function, we have developed methods that will also permit the construction of Hamiltonian densities in any field theory in which the Lagrangian density is quadratic in the first derivatives. Our Hamiltonian differs from the one obtained by Schild and Pirani in that they use Dirac's method to derive a Hamiltonian that is invariant but contains velocities, so that their canonical field equations cannot be solved with respect to the time derivatives of all canonical variables. In our formalism, the canonical equations contain no time derivatives on the right-hand sides, but the adoption of a particular Hamiltonian is equivalent to the adoption of a particular coordinate condition and gauge condition. However, once we have obtained any one Hamiltonian density, we can readily obtain any other one (and thus go over to arbitrary coordinate and gauge conditions) by combination with the other constraints of the theory in question.

Hamiltonian constraint is obtained through series of linear transformations that render trivial null vector for Legendre matrix. Second explicit gravitational Hamiltonian (after Pirani and Schild).

These first three papers preceeded the discovery of Rosenfeld's work by Bergmann's student, J. L. Anderson. All subsequent works of the Bergmann school cited Rosenfeld.

7b. Dirac chronology

- Paul Dirac presents lectures on generalized Hamiltonian dynamics in Vancouver, August 1949
  - Motivation is preservation of Poincaré covariance through parametrization of flat spacetime
  - Alfred Schild and Felix Pirani point out to Dirac applicability to general relativity
- Dirac lectures published in Canadian Journal of Mathematics in 1950 and 1951
- Pirani and Schild submit "On the quantization of Einstein's gravitational field equations" February 1950
  - Dirac, Bergmann and Brunings (1950) cited
  - First published explicit gravitational Hamiltonian (with note added in press that Bergmann group has obtained same result "using methods quite different from ours")

Dirac's breakthrough and ADM

- Dirac, "The theory of gravitation in Hamiltonian form", Proc. Roy. Soc. **A246**, 327 (1958)
  - Time derivatives of temporal components of the metric are eliminated from the Lagrangian through the subtraction of a total time derivative and a spatial divergence
  - g^{0a} are abandoned as canonical variables. Bergmann does likewise.
- ADM derive Dirac Hamiltonian in a first order Palatini variation. First to employ lapse and shift variables. (R. Arnowitt, S. Deser, C. Misner, "Canonical variables for general relativity," Phys. Rev. 117, 1597 (1960))

Dear Professor Dirac:

I have just studied your paper that appeared in the May 1 issue of the Physical Review. I am writing you, first to ask you for a reprint when they are available, but I should also like to make a few comments.

(1) The objections that Professor Lichnerowicz and I raised at the end of your lecture at Royaumont, whether or not they were valid then, certainly do not apply to the work that you have published here. Regardless of the motive of introducing the metric g_{rs} on the initial hypersurface, the canonical transformation that you first published a year ago to simplify and kill the primary constraints, is both legimate and sucessful. At this stage the total number of canonical field variables is reduced from twenty to twelve.

Excerpt of letter from Bergmann to Dirac dated October 9, 1959

(3) When I discussed your paper at a Stevens conference yesterday, two more questions arose, which I should like to submit to you: To me it appeared that because you use the Hamiltonian constraint H_L to eliminate one of the non-substantive field variables, κ , in the final formulation of the theory your Hamiltonian vanishes strongly, and hence all the final field variables, i.e. $\tilde{e}^{rs}$, $\tilde{p}^{rs}$, are "frozen" (constants of the motion). I should not consider that as a source of embarrassment, but Jim Anderson says that in talking to you he found that you now look at the situation a bit differently. Could you enlighten me? If you have no objection, I should communicate your reply to Anderson and a few other participants in the discussion.

If ~~you~~ the conditions you introduce to fix the surface are such that only one surface satisfies the conditions, then the surface cannot move at all, the Hamiltonian will vanish strongly and all dynamical variables will be frozen. However, one may introduce conditions which allow an infinity of roughly parallel surfaces. The surface can then move with one degree of freedom and there ~~will~~ must be one non-vanishing Hamiltonian that generates this motion.

I believe my condition $g_{rs} p^{rs} \approx 0$ ~~f~~ is of this second type, or maybe it also allows a more general motion of the surface corresponding roughly to Lorentz transformations. The non-vanishing Hamiltonian one would get by subtracting a divergence term from the density of the Hamiltonian.

Excerpt of response from Dirac to Bergmann, dated November 11, 1959

If the conditions that you introduce to fix the surface are such that only one surface satisfies the condition, then the surface cannot move at all, the Hamiltonian will vanish strongly and the dynamical variables will be frozen. However, one may introduce conditions which allow an infinity of roughly parallel surfaces. The surface can then move with one degree of freedom and there must be one non-vanishing Hamiltonian that generates this motion.

I believe my condition $g_{rs}p^{rs} = 0$ is of this second type, or maybe it allows also a more general motion of the surface corresponding roughly to Lorentz transformations. The non-vanishing Hamiltonian one would get by subtracting a divergence from the density of the Hamiltonian.

Postscript

Rosenfeld later denied the need to quantize gravity! He cites lack of empirical evidence. (See "Quantum theory and gravitation," 1966, reprinted in Selected Papers of Léon Rosenfeld, 1978)

Curiously, Rosenfeld was in the audience in Jablonna, Poland in 1962 when B. DeWitt presented his proof, based on a generalization of the Bohr-Rosenfeld measurability argument, that gravity must be quantized. Rosenfeld did not comment - nor was there evidently any interchange between the two on the subject (according to recollections of Cecile DeWitt.)

Conclusions

Rosenfeld made major advances in the creation of a generalized constrained Hamiltonian dynamics that ought to be recognized.

Several possible reasons can be adduced for the failure of others and Rosenfeld himself to recognize the significance of his invention of constrained Hamiltonian dynamics. Among them are

- the complexity of the mathematical procedure
- the aversion at the time to group theoretical methods
- the long-term affliction of quantum electrodynamics with unavoidable divergences
- the inadequacy of the formalism in dealing in a complete manner with general covariance
- Pauli's criticisms

It is unclear why Rosenfeld did not claim priority following the resurgence of interest in constrained Hamiltonian dynamics in the early 1970's