Le Problème du Temps en Relativité Générale

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Plan of Talk

1. Pre-Einstein concepts of time
2. Time in Einstein's Special Theory of Relativity
3. Implications of global symmetry
4. Local symmetry and the initial value problem
5. Singular Lagrangian prehistory and Leon Rosenfeld
6. Bergmann, Dirac, and the problem of time
7. General coordinate symmetry in the Hamiltonian formulation of general relativity
8. Relative ontological time in Einstein's universe?
9. Implications for quantum gravity

I - Time before Einstein

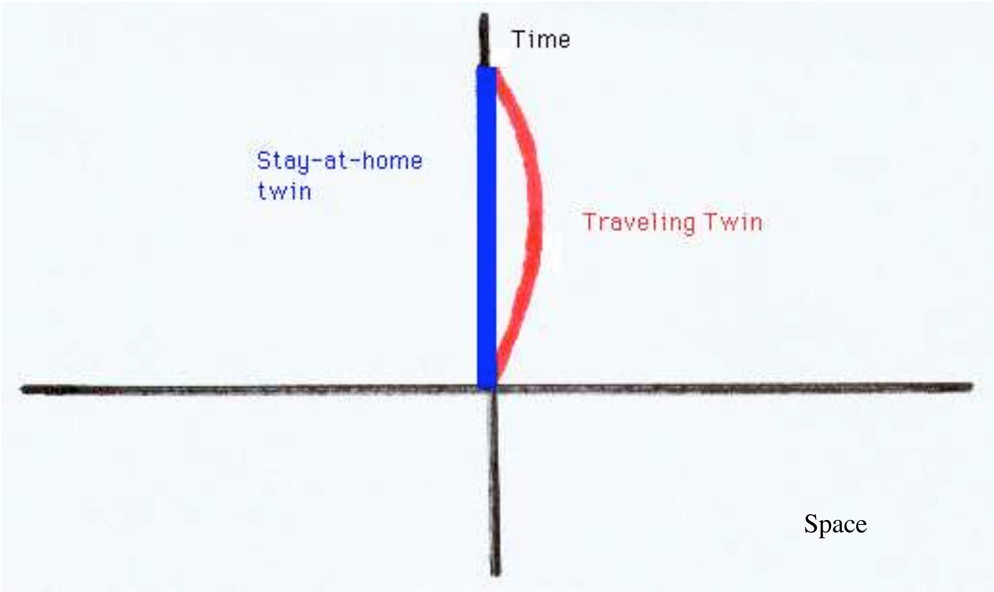
- Heraclitus (becoming) versus Parmenides (being)
- Galileo’s struggle with the continuum
- Newton’s time

I. Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external, and by another name is called duration: relative, apparent, and common time, is some sensible and external (whether accurate or unequable) measure of duration by the means of motion, which is commonly used instead of true time; such as an hour, a day, a month, a year.
…Absolute time, in astronomy, is distinguished from relative, by the equation or correction of the apparent time…The duration or perseverance of the existence of things remains the same, whether the motions are swift or slow, or none at all: and therefore this duration ought to be distinguished from what are only sensible measures thereof; and from which we deduce it, by means of the astronomical equation. The necessity of this equation, for determining the times of a phenomenon, is evinced as well from the experiments of the pendulum clock, as by eclipses of the satellites of Jupiter.

II - Time in Einstein's Special Theory of Relativity

- Observers moving at constant velocity relative to each other agree on laws of motion
- Consequence: elapsed time depends on who is measuring it!
  - Primacy of "personal time"
  - Example: spacetime diagram of traveling and stay-at-home twin

Flat spacetime geometry



Red twin ages less than blue twin

Einstein's interpretation: "spacetime distance" along red path is less than along blue path

"Straightest" spacetime path is the longest!

Square of time increment measured by traveling clocks is

$$d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

III - Global symmetry

- A symmetry transformation does not change the form of second order dynamical equations
- Example: Dynamical equation of free fall where z is the height, t the time, and g = 9.8 m/s^2

$$\frac{d^2z}{dt^2} = g$$

A particular solution:

$$z = -\frac{1}{2}gt^2$$

- Redefine time zero: $t' = t + 2$
- Then the form of the dynamical equation is unchanged:

$$\frac{d^2z}{dt'^2} = g$$

- Consequences: substitution into original solution yields a new solution:

$$z = -\frac{1}{2}g(t+2)^2 = -2g + 2gt - \frac{1}{2}gt^2$$

The symmetry group transforms the complete set of solutions into itself

- This is instance of Emmy Noether's first theorem (1918)

IV - Local symmetry and initial value problem

- Free electromagnetic field example. V is the electrostatic potential and A_x, A_y, and A_z the vector potential.

- The second order dynamical equations do not change their form under the redefinitions of the fields, where λ is an arbitrary function

$$V' = V + \frac{\partial \lambda(x,y,z,t)}{\partial t}, \qquad A_x' = A_x + \frac{\partial \lambda(x,y,z,t)}{\partial x}, \qquad \text{etc.}$$

- Implication of Noether's second theorem: solutions of dynamical equations contain an arbitrary function, so solutions are not uniquely determined by initial conditions

- But there is no loss of physical determinancy since only the electric and magnetic fields are physically observable. For example, the electric field is

$$\mathrm{E}_x = -\frac{\partial V}{\partial x} + \frac{\partial A_x}{\partial t}$$

V - Leon Rosenfeld and his pioneering work

- Born Belgium 1904
- Doctorate Liege 1926
- Research in Paris, Göttingen, Zurich 1926-1930
- Taught theoretical physics at Liege, Utrecht, Manchester, Copenhagen 1940-1974
- Collaborators and correspondents: Bohr, Pauli, de Broglie, Dirac, Heisenberg, Infeld, Klein …
- Died October 1974

Heisenberg and Rosenfeld

Rosenfeld (right, standing)
at 1933 Solvay Meeting

The initial value (Hamiltonian) formulation of electromagnetism

- Required to take canonical route to quantization of the electromagnetic field
- Must rewrite equations of motion so as to contain only first derivatives with respect to time. This is done by defining new variables, the "momenta", in terms of the velocities.

  For example, in the case of the object in free fall, let $p = \frac{dz}{dt}$

  then this definition plus the equation of motion $\frac{dp}{dt} = -g$

  become the Hamiltonian equations of motion. These equations of motion are determined by a "Hamiltonian"

  $$H = gz + \frac{1}{2}p^2$$

- But there is a problem with electromagnetism. One of the canonical momenta (the one associated with the time derivative of the electrostatic potential) vanishes identically!

Proposals for dealing with vanishing momentum:

Heisenberg/Pauli formalism (1929-1930)

• Add non-gauge invariant term to Lagrangian (destroys local symmetry)

• Or set *V = constant* (destroys manifest Lorentz invariance)

Pauli: “Ich warne Neugierige”

Rosenfeld’s debt to Pauli: “*As I was investigating these relations in the especially instructive example of gravitation theory, Professor Pauli helpfully indicated to me the principles of a simpler and more natural manner of applying the Hamiltionian procedure in the presence of identities*” (My translation from Rosenfeld’s 1930 paper)

Rosenfeld's formal constraint analysis in "On the quantization of wave fields", Annalen der Physik 1930

Application to quantum electrodynamics addressed in "La théorie quantique des champs", Annales de l'Institut Henri Poincaré, 1932

Zur Quantelung der Wellenfelder

Von L. Rosenfeld

Einleitung

Wesentliche Fortschritte in der Formulierung der allgemeinen Quantengesetze der elektromagnetischen und materiellen Wellenfelder haben neuerdings Heisenberg und Pauli[1]) erzielt, indem sie die von Dirac erfundene „Methode der nochmaligen Quantelung" systematisch entwickelten. Neben gewissen sachlichen Schwierigkeiten, die viel tiefer liegen, trat dabei eine eigentümliche Schwierigkeit formaler Natur auf: der zum skalaren Potential kanonisch konjugierte Impuls verschwindet identisch, so daß die Aufstellung der Hamiltonschen Funktion und der Vertauschungsrelationen nicht ohne weiteres gelingt. Zur Beseitigung dieser Schwierigkeit sind bisher drei Methoden vorgeschlagen worden, die zwar ihren Zweck erfüllen, aber doch schwerlich als befriedigend betrachtet werden können.

1. Die erste Heisenberg-Paulische Methode ist ein rein analytischer Kunstgriff.[2]) Man fügt zur Lagrangefunktion gewisse Zusatzglieder hinzu, die mit einem kleinen Parameter ε multipliziert sind und bewirken, daß der obenerwähnte Impuls nicht mehr verschwindet. In den Schlußresultaten muß man dann zum Limes $\varepsilon = 0$ übergehen. Die ε-Glieder führen aber zu unphysikalischen Rechenkomplikationen[3]) und zerstören die charakteristische Invarianz der Lagrangefunktion gegenüber der Eichinvarianzgruppe.

2. Die zweite Heisenberg-Paulische Methode[4]) benutzt hingegen wesentlich diese Invarianz. Dem skalaren Potential

1) W. Heisenberg u. W. Pauli, Ztschr. f. Phys. 56. S. 1. 1929; ebenda 59. S. 168. 1930. Im folgenden mit H. P. I bzw. II zitiert.

2) H. P. I, S. 24—26, 30ff.

3) Vgl. L. Rosenfeld, Ztschr. f. Phys. 58. S. 540. 1929.

4) H. P. II.

Rosenfeld's formal constraint analysis in "On the quantization of wave fields", Annalen der Physik 1930

- Local symmetries always lead to
  - non-unique evolution in time
  - constraining relations among variables and associated momenta
  - Hamiltonian (from which equations of motion are determined) constructed using the constraints
  - vanishing of Hamiltonian if, as in general relativity, the equations of motion take the same form for arbitrary choices of the time coordinate
- Rosenfeld was first to consider how to implement local symmetry-induced transformations on Hamiltonian variables
- Rosenfeld's dynamical model - gravitation with a charged spinorial

Dirac field source

Origins of the model

- Weyl/Fock coupling of Dirac field with gravity - 1929
- Tetrads and Weyl's reinterpretation of gauge symmetry
  See analyses by Scholz (physics/0409158) and Straumann (hep-ph/0509116)

VI -The symmetry of Einstein's equations under general coordinate transformations

Square of time increment measured by traveling clocks in Einstein's general theory is

$$d\tau^2 = N^2 c^2 dt^2 - \sum_{a,b=1}^{3} g_{ab}\left(N^a c dt + dx^a\right)\left(N^b c dt + dx^b\right), \text{ where } x^\mu := \{ct, x, y, z\}$$

lapse

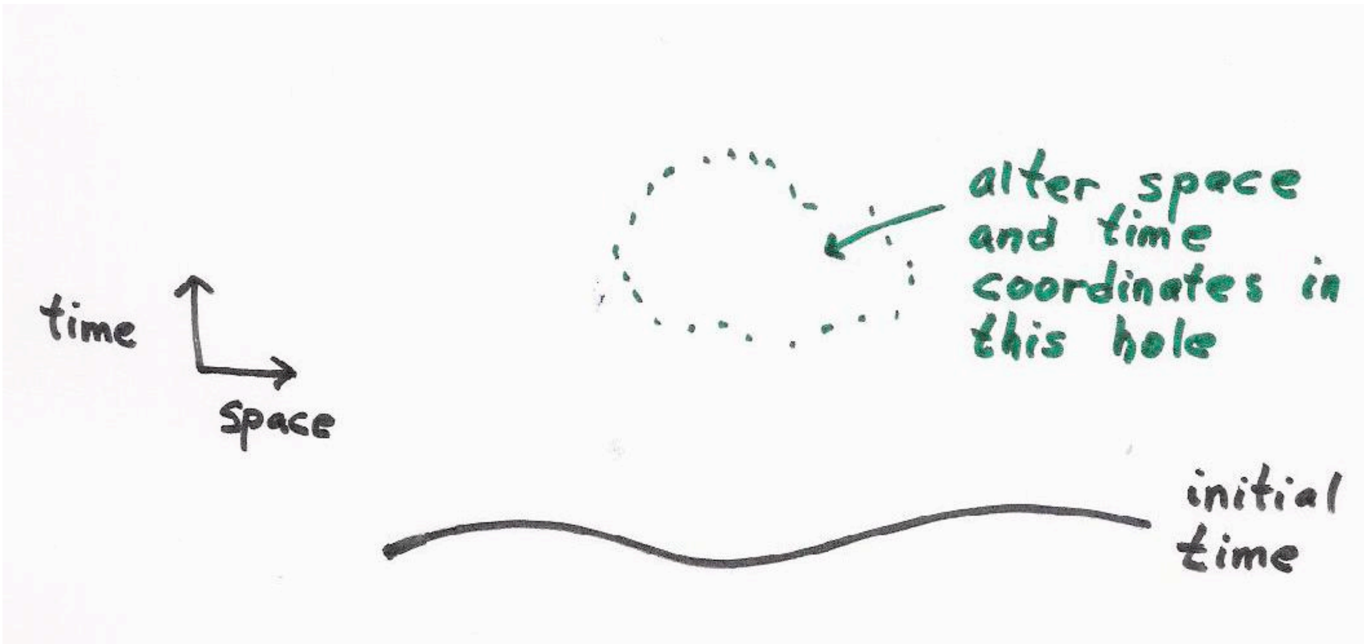
spatial metric

shift

- Einstein's second order equations of motion take the same form for <u>all</u> choices of coordinates *t, x, y,* and *z*. There is no preferred way of assigning spatial or temporal coordinates in general relativity
- As a consequence of general covariance the full set of solutions of Einstein's second order equations transforms into itself under <u>arbitrary</u> changes of coordinates

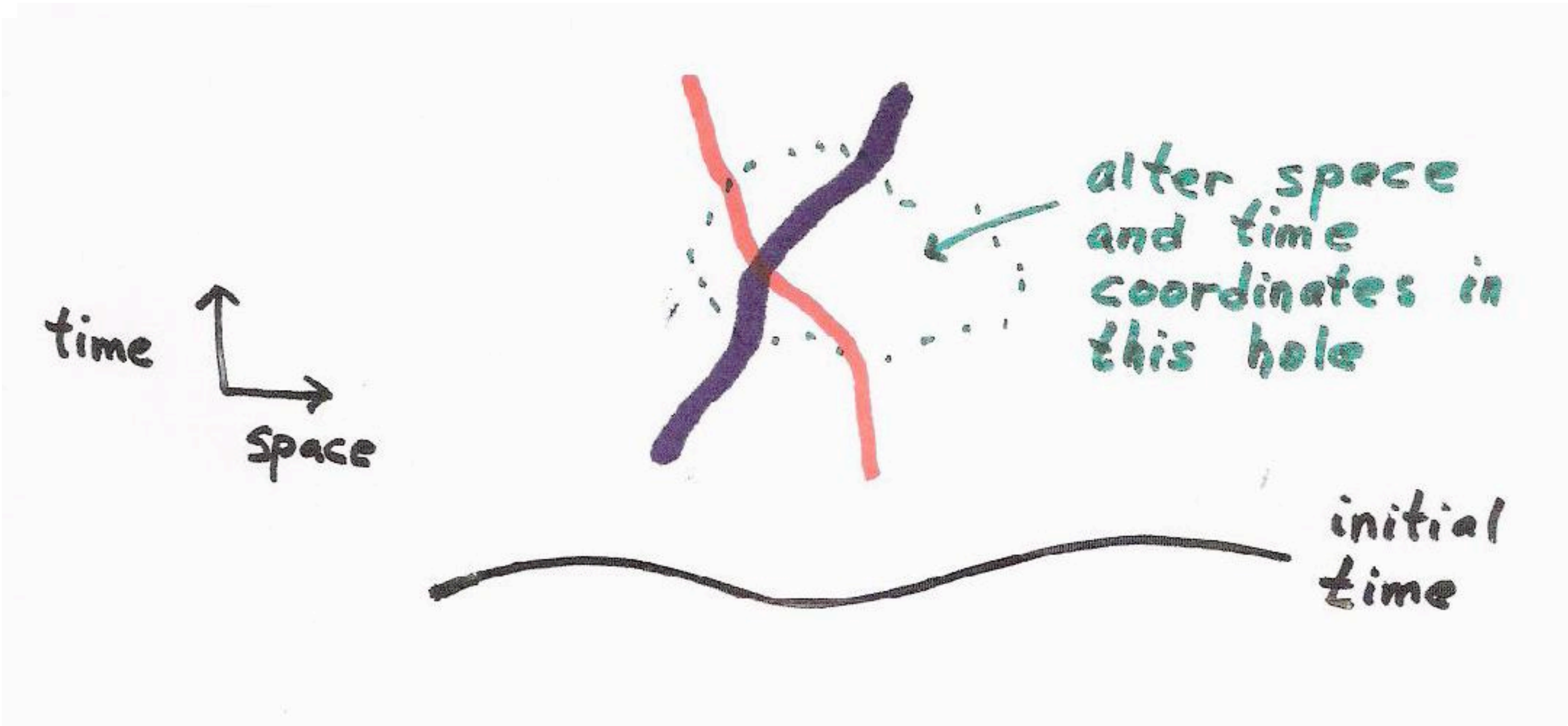
$x'^\mu(x)$. In particular, $t' = t +$ constant, is a symmetry of Einstein's equations, but not of the first order Hamiltonian equations

Einstein's initial rejection of general coordinate symmety, and his resolution: the "hole argument"



Einstein (1913): change of space and time coordinates in the hole produces changes in "physical" quantities in the hole, but no changes at the initial time. But since the dynamical equations have the same form under arbitrary changes in coordinates (general covariance), the transformed quantities represent new solutions - but with the same starting assumptions. Looks like breakdown of determinism.

Einstein finally in 1915 embraced coordinate symmetry (general covariance) by proclaiming that only material coincidences were real. Particle collisions identify points (events) in spacetime



Initial value formulation of general relativity

We now have a way - following the pioneering work of Rosenfeld, Bergmann, and Dirac - of tracking the evolution from an initial instant for any choice we wish to make for a temporal coordinate

Bergmann and Komar (1972), following up on the work of Paul Dirac (1958), made the first step in understanding how general coordinate symmetry is preserved in the initial value (Hamiltonian) version of general relativity

Brief Bergmann biography

- Born Berlin-Charlottenburg 1915
- Mother Dr. Emmy Bergmann moved with children to Freiburg 1922 - she and sister emigrated to Israel 1935
- Father Dr. Max Bergmann 1921 - 1933 head of Institut für Lederforschung, Dresden (now Max Bergmann Zentrum für Biomaterialen)
- Prague, Charles University degree 1936
- Einstein Assistant 1936 - 1941: unified field theory
- Syracuse University 1947 - 1982
- Died October 2002

Excerpt of letter from Peter Bergmann to Nathan Rosen, dated September 26, 1973

Dirac is perhaps the last of the really great pioneers that created today's physics. Though he may not be able to last through a heavy conference schedule, he will take in a few papers a day every day for a week, and he will make very helpful and acute comments on occasion. His presence will, of course, lend prestige to GRG7, but he will be a real asset as a physicist. Having through an extended period wrestled with the same problems that he succeeded in solving - a viable Hamiltonian version of general relativity, I have the profoundest respect for his genius, second only (in my personal experience) to Einstein. I think that you should act soon.

VII -The "problem of time" in general relativity

If evolution in time (global translation in time) were a symmetry in the Hamiltonian version of general relativity, and observables were understood to be invariant under symmetry transformations, the all observables in general relativity would be constants of the motion!

Excerpt of letter from Bergmann to Dirac dated October 9, 1959:

Dear Professor Dirac:

I have just studied your paper that appeared in the May 1 issue of the
Physical Review. I am writing you, first to ask you for a reprint when
they are available, but I should also like to make a few comments.

(1) The objections that Professor Lichnerowicz and I raised at the
end of your lecture at Royaumont, whether or not they were valid then,
certainly do not apply to the work that you have published here. Regard-
less of the motive of introducing the metric g_{ik} on the initial hyper-
surface, the canonical transformation that you first published a year ago to
simplify and kill the primary constraints, is both legimate and sucess-
ful. At this stage the total number of canonical field variables is
reduced from twenty to twelve.

(3) When I discussed your paper at a Stevens conference yesterday, two more questions arose, which I should like to submit to you: To me it appeared that because you use the Hamiltonian constraint H_L to eliminate one of the non-substantive field variables, κ , in the final formulation of the theory your Hamiltonian vanishes strongly, and hence all the final field variables, i.e. $\tilde{e}^{rs}, \tilde{p}^{rs}$, are "frozen" (constants of the motion). I should not consider that as a source of embarrassment, but Jim Anderson says that in talking to you he found that you now look at the situation a bit differently. Could you enlighten me? If you have no objection, I should communicate your reply to Anderson and a few other participants in the discussion.

If ~~you~~ the conditions you introduce to fix the surface are such that only one surface satisfies the conditions, then the surface cannot move at all, the Hamiltonian will vanish strongly and all dynamical variables will be frozen. However, one may introduce conditions which allow an infinity of roughly parallel surfaces. The surface can then move with one degree of freedom and there ~~will~~ must be one non-vanishing Hamiltonian that generates this motion.

I believe my condition $g_{rs}p^{rs} \approx 0$ ~~#~~ is of this second type, or maybe it allows also a more general motion of the surface corresponding roughly to Lorentz transformations. The non-vanishing Hamiltonian one would get by subtracting a divergence term from the density of the Hamiltonian.

Excerpt of response from Dirac to Bergmann, dated November 11, 1959

If the conditions that you introduce to fix the surface are such that only one surface satisfies the condition, then the surface cannot move at all, the Hamiltonian will vanish strongly and the dynamical variables will be frozen. However, one may introduce conditions which allow an infinity of roughly parallel surfaces. The surface can then move with one degree of freedom and there must be one non-vanishing Hamiltonian that generates this motion.

I believe my condition $g_{rs}p^{rs} = 0$ is of this second type, or maybe it allows also a more general motion of the surface corresponding roughly to Lorentz transformations. The non-vanishing Hamiltonian one would get by subtracting a divergence from the density of the Hamiltonian.

VIII. General coordinate symmetry in the Hamiltonian formulation of general relativity

- A symmetry must transform the complete set of solutions of Einstein's equations into itself
- To accomplish this feat the lapse and shift must be retained as dynamical variables
- Because of the singular nature of the Lagrangian of general relativity, functions of the time derivatives of the lapse and shift do not map onto functions on phase space (the space of field variables and corresponding momenta) - but variations of the lapse and shift under general coordinate transformations $x'^{\mu}(x)$ do depend on these time derivatives.
- Pons, Salisbury and Shepley (1997-2001) showed that the underlying initial value (Hamiltonian) symmetry is relational in the sense that the symmetries depend not only on arbitrary spacetime functions - but necessarily also on the physical gravitational field. Hamiltonian symmetry transformations are of the form

$$x'^{\mu} = x^{\mu} + n^{\mu}\xi^{0}(x;g_{bc}) + \sum_{a=1}^{3} \delta_{a}^{\mu}\xi^{a}(x;g_{bc}),$$

$$\text{where } n^{\mu} = \left\{\frac{1}{N}, -\frac{N^{a}}{N}\right\}$$

Note: This is the first indication that a knowledge of time requires reference to the gravitational field

- Pons and Salisbury (2005) explained how to construct a univocal relational time, exploiting the newly discovered Hamiltonian symmetry. An “intrinsic” time is defined using an appropriate function of physical fields.
- Intrinsic time is a relational time. A univocal correlation is established between the value of the chosen intrinsic time function and the value of all the other physical variables
- There are three equivalent ways of constructing phase space observables - functions of phase space variables that are invariant under the action of the new symmetry group
  - Given solutions in any coordinate system, undertake a change in coordinates to the intrinsic coordinate system. Every variable expressed in this coordinate system is an observable
  - Given solutions in any coordinate system, “gauge transform” under the symmetry transformation to the solution for which the function has the simple time dependence t
  - Impose the gauge condition in the Hamiltonian formalism: t = the chosen function of phase space variables

IX - Relationship of intrinsic coordinates to the programs of Kuchàr, Rovelli and Barbour

- Kuchàr does not recognize the existance of a Hamiltonian symmetry group that includes changes in time. His "bubble time" advancement in time does not form a group
- Rovelli's program of partial and complete observables could be related to intrinsic coordinates, but not in the way it is currently applied. The apparent sole condition to be satisfied by a Rovelli "clock" variable is that it grow monotonically with coordinate time. But the "parameter clock time" is not necessarily a coordinate time for all physically inequivalent solutions. Rovelli is careful not to describe "parameter evolution" as physical time evolution. This distinction appears to be necessary since if the parameter time were identified as a coordinate time in general the resulting evolution will not satisfy Einstein's equations.
- Our program results in invariant variables (constructed in terms of the phase space variables) that exhibit a time evolution satisfying Einstein's equations
- Barbour's point of view seems to be based on the now discounted claim that global translation in time is a Hamiltonian symmetry, with the result that all observables will be constants of the motion

VI - A cosmological example of relative ontological(?) time

- Isotropic expanding universe containing a massless scalar field, with two gravitational (metric) variables, the spatial metric (expansion factor) *a* and the lapse function *N*
- The Hamiltonian model is symmetric under the infinitesima time transformation

$$t' = t - \frac{\xi(t)}{N(t)}$$

- Choose the square of the expansion factor as the intrinsic time since it increases monotonically with coordinate time
- The model fixes a unique correlation between the value of a^2 and the value of the scalar field
- It can be shown explicitly that the resulting fields are invariant under the group of transformations given above - thus we have true evolution in intrinsic time, but only when there is stuff in the universe!

VII - Implications for quantum gravity

- In the loop approach to quantum gravity a^2 can take only certain discrete values, determined in terms of the Planck time (about 10^{-43} seconds)

- Although most researchers in the field are satisfied that no notion of temporal evolution need be present in the Planckian era, we maintain that one can sensibly construct a generalized Schroedinger quantum time stepping.

- Most of the quantum relativity community is still convinced that quantum time is “frozen”, yet most also recognize the possibility of non-trivial evolution in “parameter” time.

Generalized time-dependent Schrödinger equation (gr-qc/0702132)

Use discrete time eigenvalues from loop gravity

$$t_k = \frac{k}{6}, \qquad k = 0,1,2,\ \ldots$$

Let $\left|\psi\left(\phi, t_{k+1}\right)\right\rangle = \left(1 - \frac{i\Delta t H}{\hbar}\right)\left|\psi\left(\phi, t_k\right)\right\rangle = \left(1 - \frac{9i}{2\hbar^2\left(k+1\right)}\right)\left|\psi\left(\phi, t_k\right)\right\rangle$