General Covariance, Observables, and Emergent Time

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Canonically realizable induced coordinate transformation symmetries applied to isotropic cosmology with massless scalar source, induced by lapse-dependent infinitesimal time reparameterization

$$t' = t + \frac{\xi(t)}{N(t)}$$

Generator is $G(\xi) = p_N \dot{\xi} + N\left(-\frac{\pi p_a^2}{3a} + \frac{p_\phi^2}{2a^3}\right)\xi := p_N\dot{\xi} + H\xi$

So finite one-parameter family of gauge transformations are implemented by

$$\exp\left(\{-, sG(\xi)\}\right)$$

Note that Hamiltonian is distinct from gauge generator: $H_\lambda = NH + \lambda p_N$

Intrinsic time is a time-dependent gauge choice. Observables (invariants under reparameterizations) are simply dynamical phase space variables evaluated in the chosen intrinsic coordinate system. They may be obtained either through an active finite gauge transformation generated by G, or through a passive reparameterization

Intrinsic time must be a spacetime scalar function *T* of the dynamical variables. Need to find descriptor for finite gauge transformation that actively transforms *T* to t, i.e.

$$t = \exp\left(\{-, G(\xi)\}\right) T := T + \{T, G(\xi)\} + \frac{1}{2}\left\{\{T, G(\xi)\}, G(\xi)\right\} + \cdots$$

Task is easily accomplished with constraint modified with a factor so as to satisfy

$$\{T, \overline{H}\} \approx 1 \qquad \text{where} \qquad \overline{H} := \mathcal{B}H$$

In terms of this modified constraint the gauge generator becomes

$$G(\bar{\xi}) = p_N \mathcal{B}\dot{\bar{\xi}} + N\{\mathcal{B}, H\} p_N \bar{\xi} + \overline{H}\bar{\xi}$$

Then the required descriptor is simply $\bar{\xi} = t - T$

To simplify the dynamics and gauge symmetry define new variables

$$u_{\pm} := \frac{1}{\sqrt{2}}\left(\ln\left(\frac{3}{2\pi}\right)^{1/3} + \ln a \pm \frac{9}{2}\frac{2\pi}{3}\phi\right) \qquad n := \frac{9}{2}N$$

The Hamiltonian constraint becomes $H = -\exp\left(-u_{+} - u_{-}\right) p_{-} p_{+}$

with equations of motion (assuming $p_{-} \approx 0$)

$$\dot{u}_{+} \approx 0 \quad \dot{u}_{-} = n \exp\left(u_{+} + u_{-}\right) p_{+} \quad \dot{p}_{+} \approx 0$$

Choose (among the infinite number of possibilities) the intrinsic time $t = u_{-}$. Then

$$\mathcal{B} = -\frac{1}{p_{+}} \exp\left(u_{+} + u_{-}\right) \text{ and } \overline{H} = p_{-}$$

Thus the gauge generator becomes

$$G(\overline{\xi}) = -p_N \frac{1}{p_{+}} \exp\left(u_{+} + u_{-}\right) \dot{\overline{\xi}} + N p_N \overline{\xi} + \overline{H} \xi$$

Just a reminder: $u_{\pm} := \frac{1}{\sqrt{2}}\left(\ln\left(\frac{3}{2\pi}\right)^{1/3} + \ln a \pm \frac{9}{2}\frac{2\pi}{3}\phi\right)$

As an example consider the active transformation of the lapse

$$N_I = N + \{N, G(\xi)\} + \frac{1}{2}\{\{N, G(\xi)\}, G(\xi)\} + \cdots$$

$$= -\frac{1}{p_+}\exp(u_+ + u_-)\exp(t - u_-) = -\frac{1}{p_+}\exp(u_+)\exp(t)$$

Note well that the phase space dependence of the descriptor is substituted only after Poisson brackets are computed. However, it does turn out to be possible to obtain invariants as a limit of a family of canonical transformations.

This is an example of a general result in which invariants appear as coefficients in Taylor series expansions in powers of intrinsic temporal and spatial coordinates. (The coordinates are themselves invariant.)

Just a reminder: $$u_\pm := \frac{1}{\sqrt{2}}\left(\ln\left(\frac{3}{2\pi}\right)^{1/3} + \ln a \pm \frac{9}{2}\frac{2\pi}{3}\phi\right)$$

A curious result: there exists an intrinsic time for which the only variable is the time itself. This is easily seen by making the change of variables

$$x_\pm := \exp\left(-u_\pm\right)$$

Resulting in $$H = -p_+ p_-$$

Lesson: choice of intrinsic (relative) time always corresponds to a specific choice of lapse and shift. If for example we choose the scalar field as the intrinsic time a specific scalar field dependence will appear in the expressions for the lapse and shift. This will lead to specific scalar field dependence - effective potentials - if one's starting point is the Friedmann and Raychaudhuri equations.