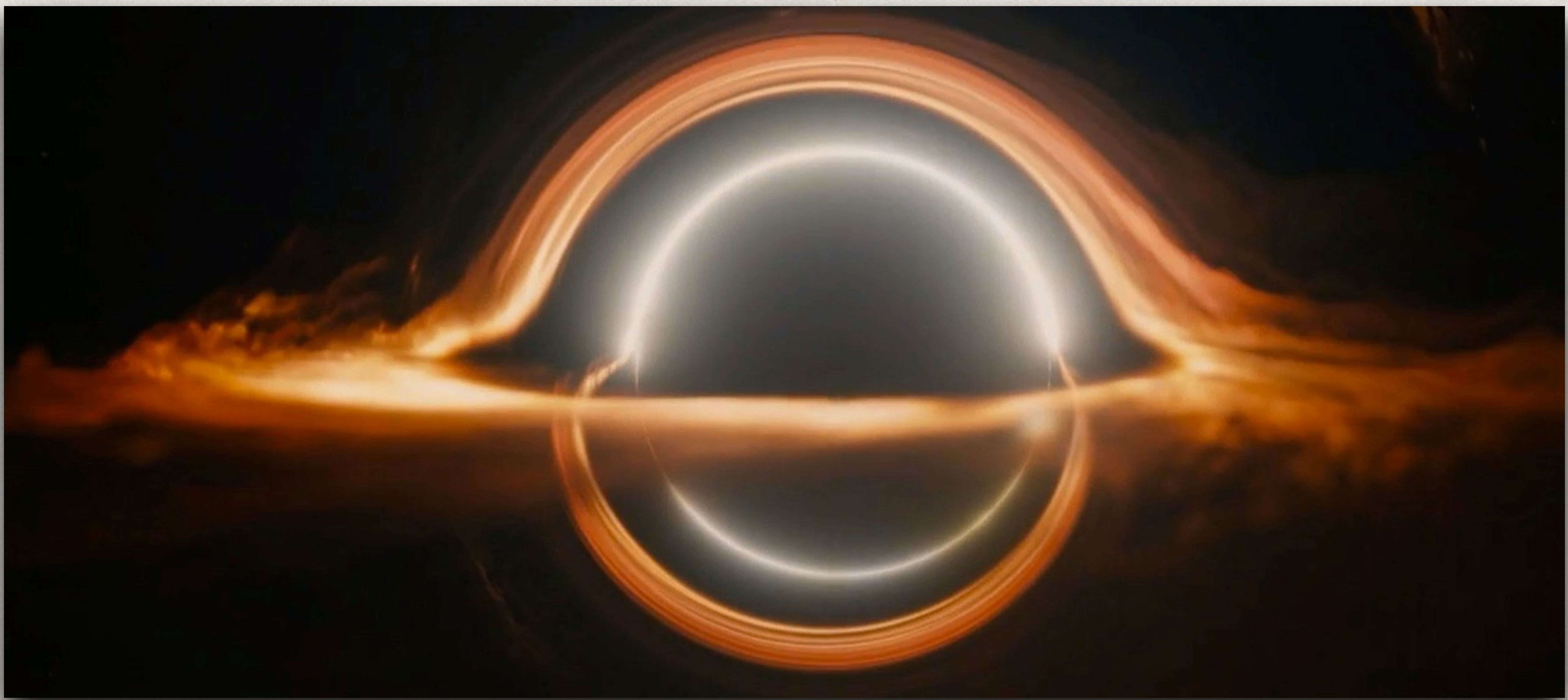


[Double Negative]

Tidal deformation of black holes and neutron stars

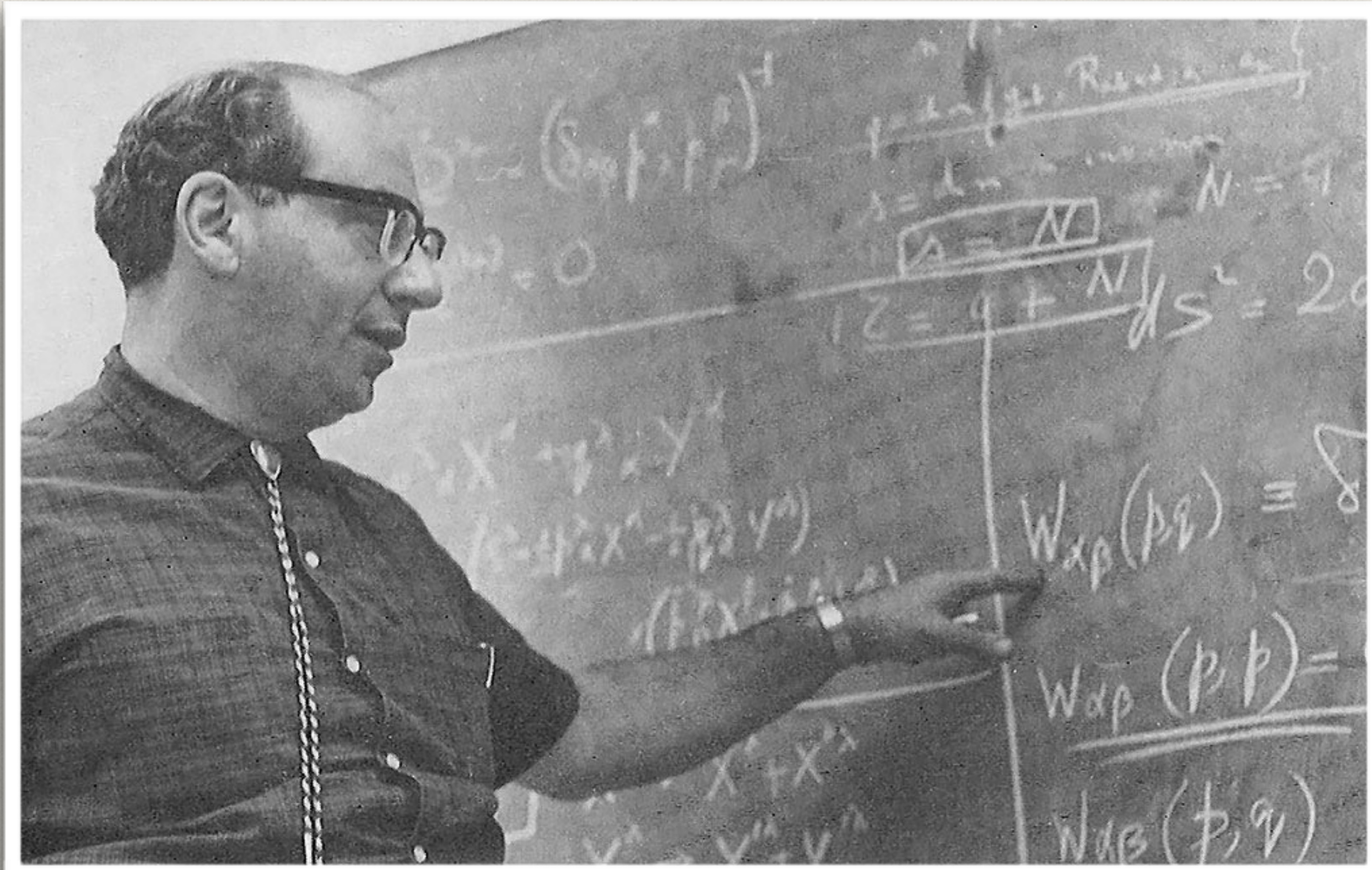
Eric Poisson, University of Guelph

Ivor Robinson Symposium, Dallas, May 7-9, 2017

UNIVERSITY of GUELPH

CHANGING LIVES IMPROVING LIFE

Ivor Robinson

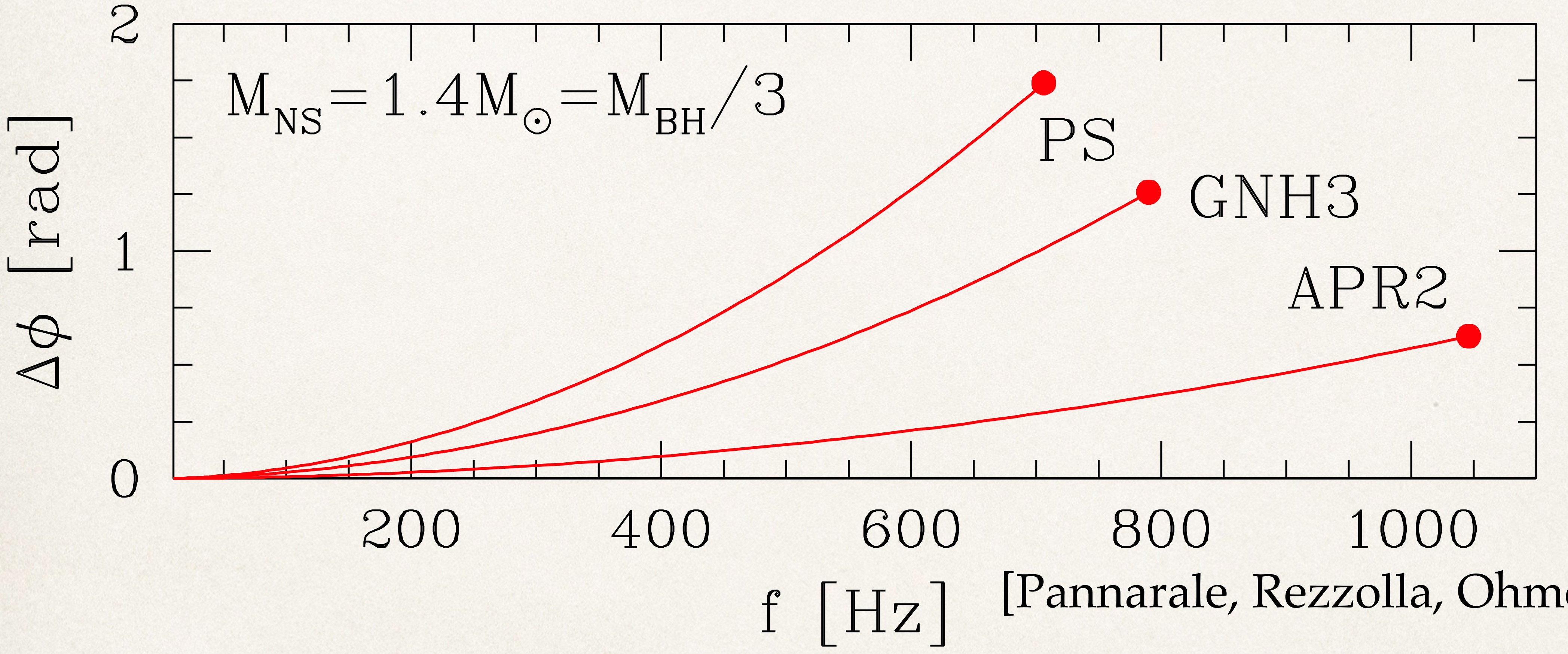


"I owe Ivor a great deal. We first met at a conference in 1963, and he at once invited me (an unknown greenhorn, all expenses paid!) to the first Texas Symposium in Dallas. That was the most exciting event I ever attended. It opened up a completely new direction for my research and changed my life."

Werner Israel (April 6, 2017)

Context

- Tidal deformations of neutron stars affect their orbital motion, and this can have a measurable impact on the emitted gravitational waves [Flanagan and Hinderer (2008)]

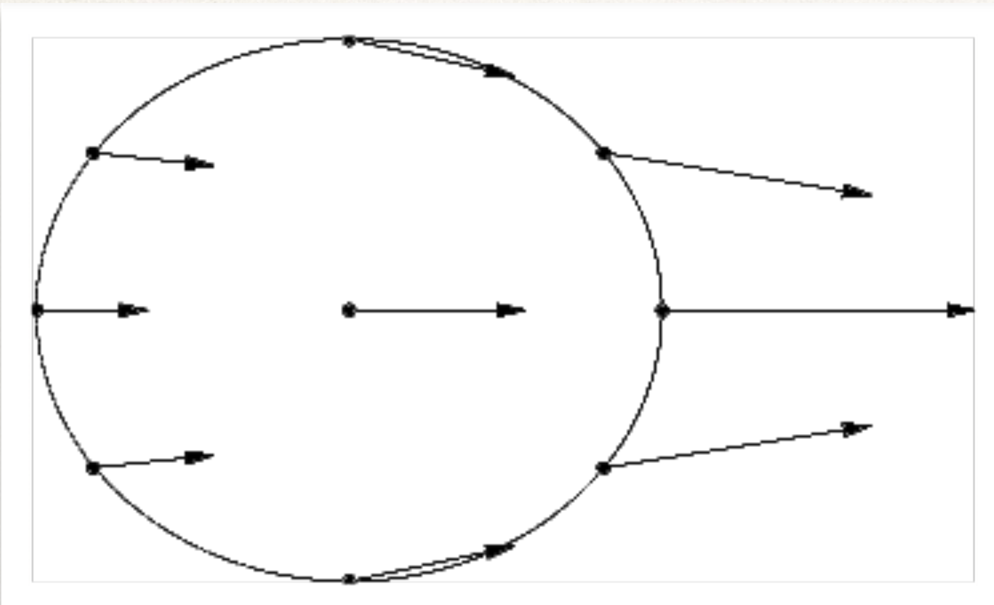


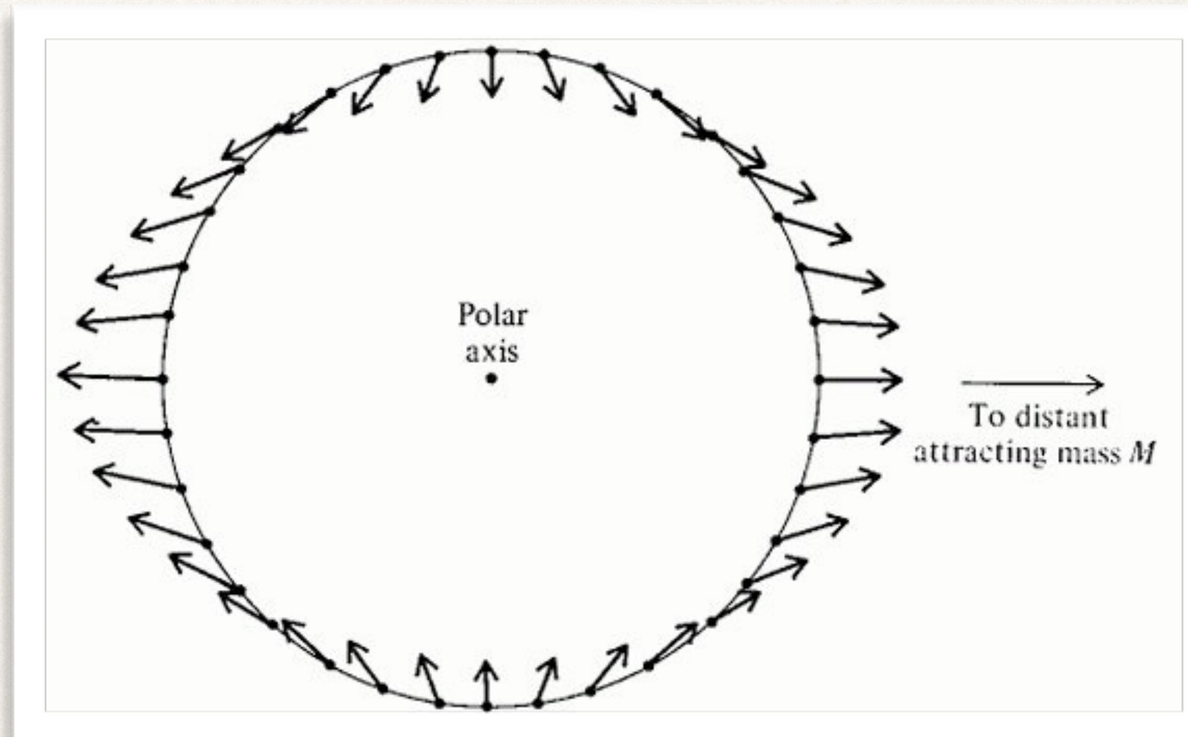
[Pannarale, Rezzolla, Ohme, Read (2011)]

- Gravitational-wave measurement of inspirals can reveal information about the internal structure of neutron stars

Newtonian theory of tides

Tidal forces are produced by inhomogeneities in the gravitational field produced by a remote body





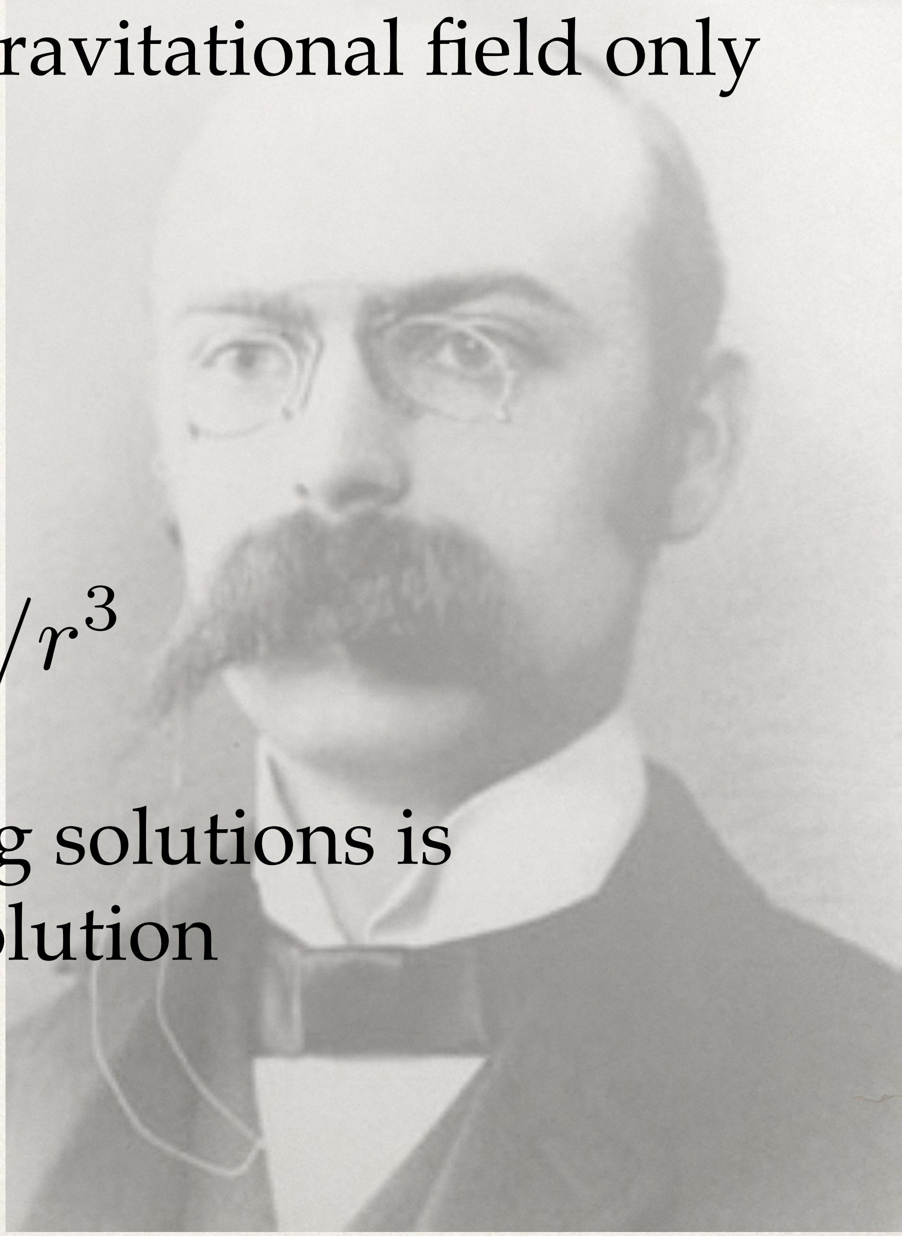
$$U_{\text{ext}} = U_{\text{ext}}(0) + g_a x^a - \frac{1}{2}\mathcal{E}_{ab} x^a x^b + \cdots$$

$$Q_{ab} = \int \rho\left(x_a x_b - \tfrac{1}{3} r^2 \delta_{ab}\right) dV$$

$$Q_{ab} = -\frac{2}{3} k_2 R^5 \mathcal{E}_{ab}$$

The Love number k_2 encapsulates the details of the body's internal structure

External physics



- The physics external to the body involves the gravitational field only

$$\nabla^2 U = 0$$

  - Growing solution (tidal field): $U \sim \mathcal{E} r^2$
  - Decaying solution (tidal response): $U \sim k_2/r^3$
  - The precise mixture of growing and decaying solutions is determined by matching with the internal solution
  - This determines the **Love number *k_2***

Internal physics

- The physics inside the body involves gravity and the matter variables
- A simple matter model is a perfect fluid (density, pressure, velocity)
- The effect of the tidal forces can be determined in perturbation theory
- All variables can be decomposed in normal modes, and each mode behaves as a simple harmonic oscillator

$$\ddot{\xi} + \omega^2 \xi = f$$

external tidal force

restoring force (pressure gradients)

- For a slowly changing tidal field: $\xi = f/\omega^2$ (perturbed equilibrium)

Tides in GR

- There are two types of tidal fields in general relativity: **gravitoelectric** and **gravitomagnetic**

  - Given a timelike vector field, the Weyl tensor can be decomposed as

$$\mathcal{E}_{\alpha\beta} = u^\mu u^\nu C_{\alpha\mu\beta\nu}$$
$$\mathcal{B}_{\alpha\beta} = \frac{1}{2} u^\mu u^\nu \epsilon_{\mu\alpha\gamma\delta} C^{\gamma\delta}{}_{\beta\nu}$$

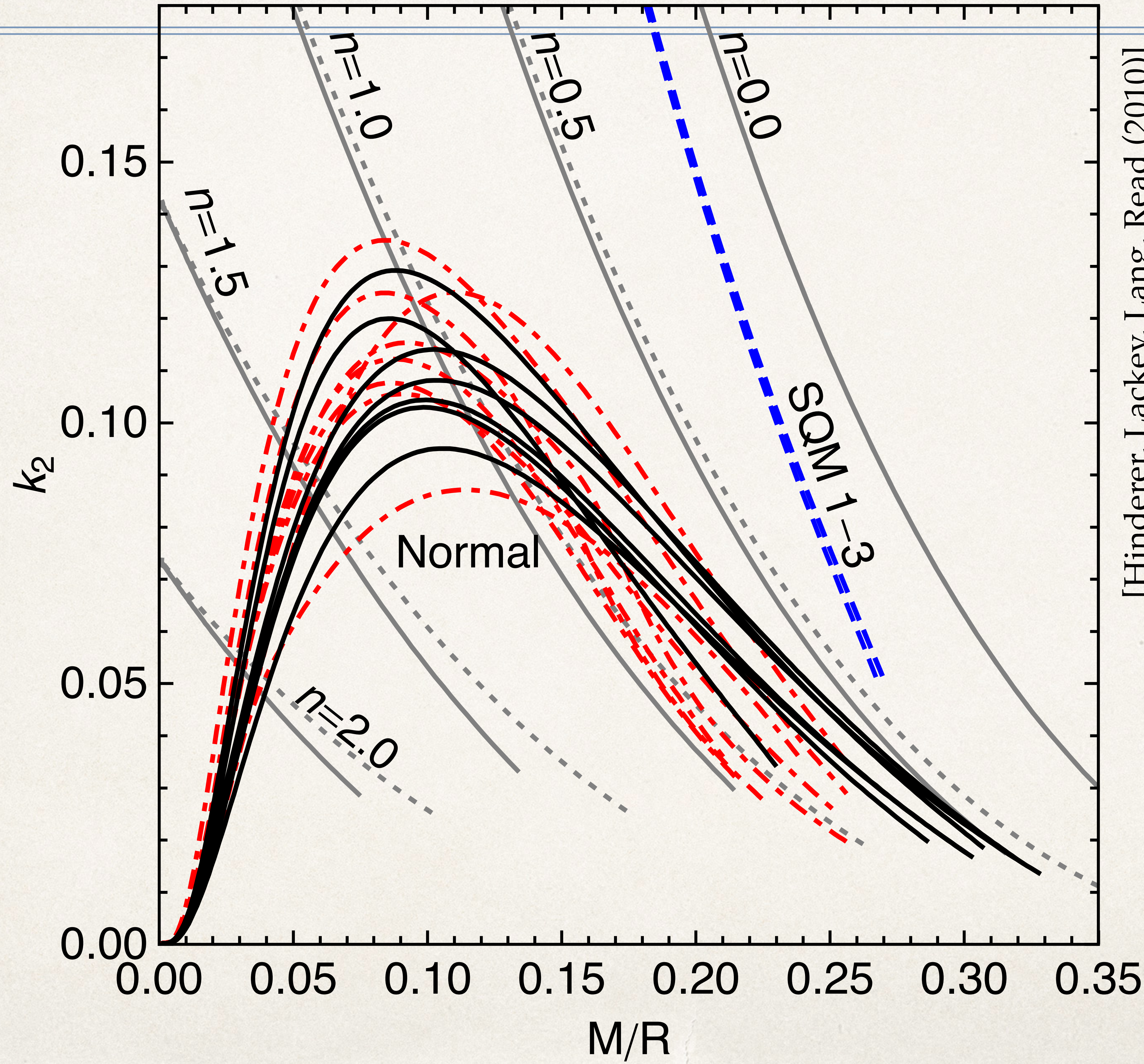
  - Much of post-Newtonian gravity is captured by scalar and vector potentials,

$$\nabla^2 U = -4\pi\rho \qquad \nabla^2 U_a = -4\pi\rho v_a$$

Nonrotating bodies

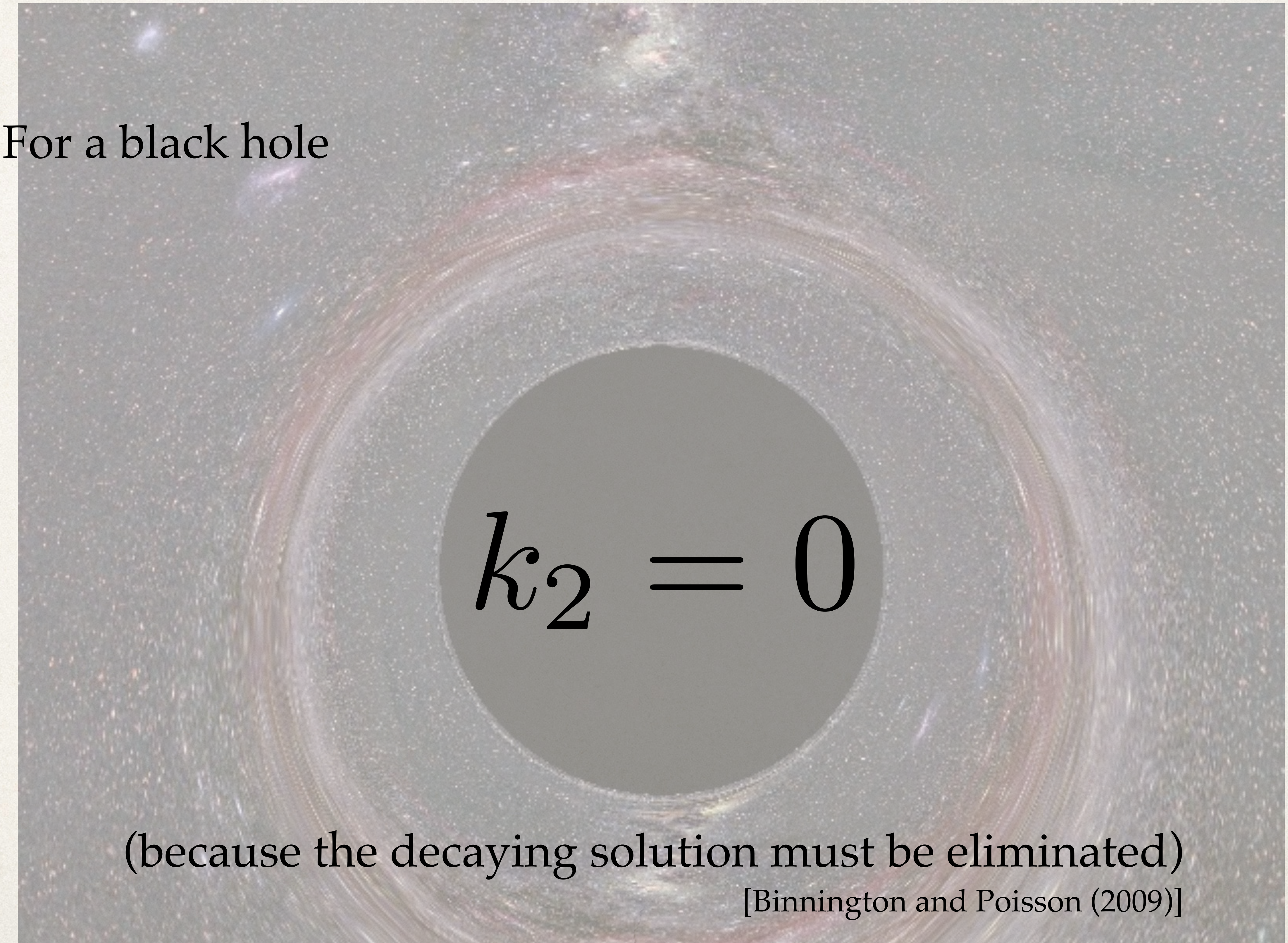
- **External physics:** perturbed Schwarzschild metric
  - Growing solution: regular at $r=2M$, diverges at infinity
  - Decaying solution: diverges at $r=2M$, vanishes at infinity
  - The precise mixture of growing and decaying solutions is determined by matching with the internal solution
  - This determines the **relativistic Love number k_2**
- **Internal physics:** perturbed fluid and internal metric

Relativistic Love number



[Hinderer, Lackey, Lang, Read (2010)]

Relativistic Love number



- For a black hole

$$k_2 = 0$$

(because the decaying solution must be eliminated)

[Binnington and Poisson (2009)]

Rotating black hole

- The tidal deformation of a rotating black hole can be computed with the help of the Teukolsky equation [O'Sullivan and Hughes (2014, 2016); Penna, Hughes, O'Sullivan (2017)]

- When $\chi = J/M^2 \ll 1$ the rotational deformation of the black hole can be neglected, and the tidal deformation incorporates all effects associated with the dragging of internal frames [Poisson (2015)]

- The rotation produces a **phase lag** (not lead) between the horizon's tidal bulge and the tidal field: $(\Omega_{\rm tide} \ll \Omega_{\rm H})$

$$\varphi_{\rm bulge} = \big(\Omega_{\rm tide} - \Omega_{\rm H}\big) v - \frac{2}{3}\chi$$

Rotating neutron star

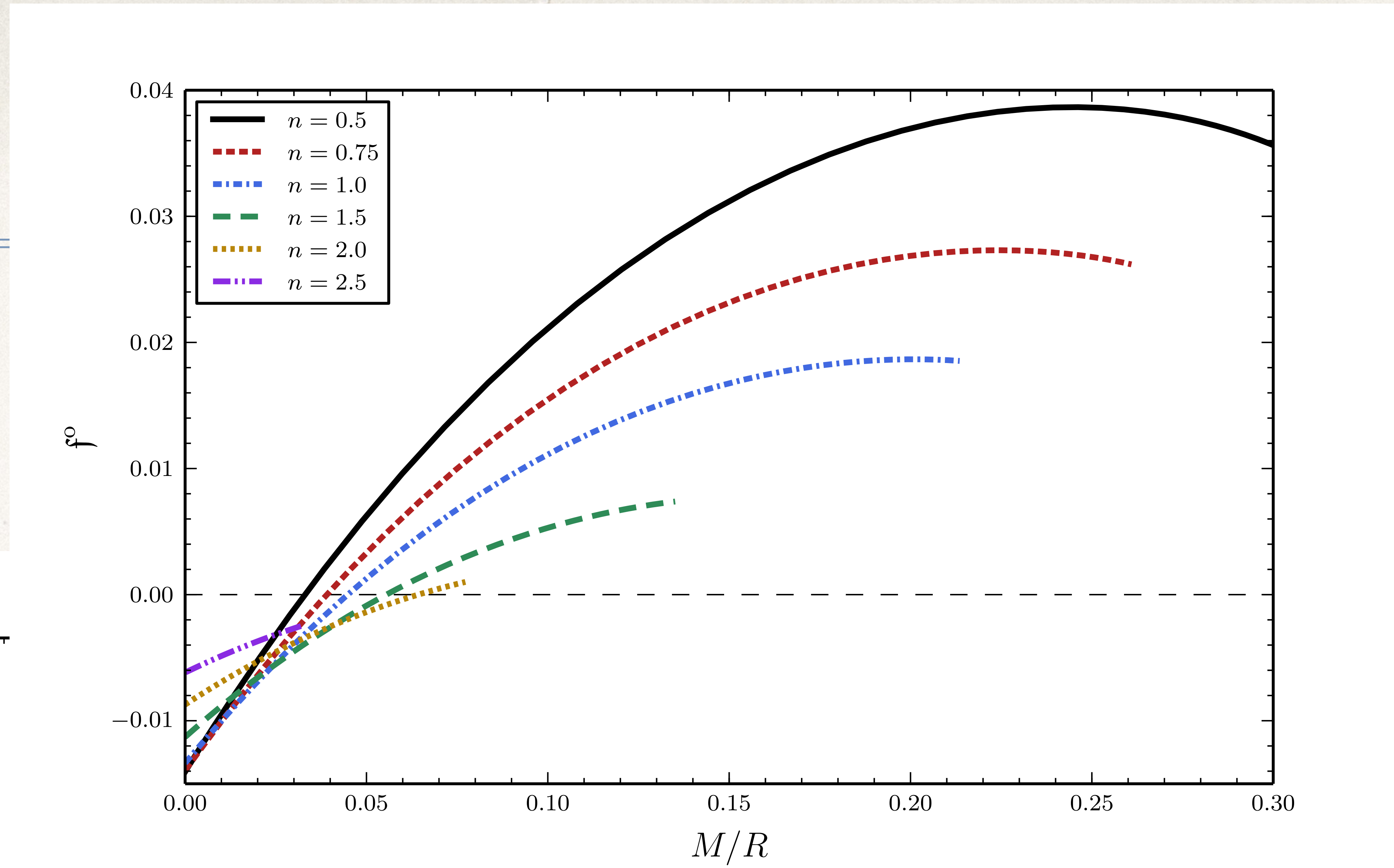
- The coupling between rotation and tidal field requires the introduction of new (octupolar) Love numbers

  [Landry and Poisson (2015); Pani, Gualtieri, Ferrari (2015)]

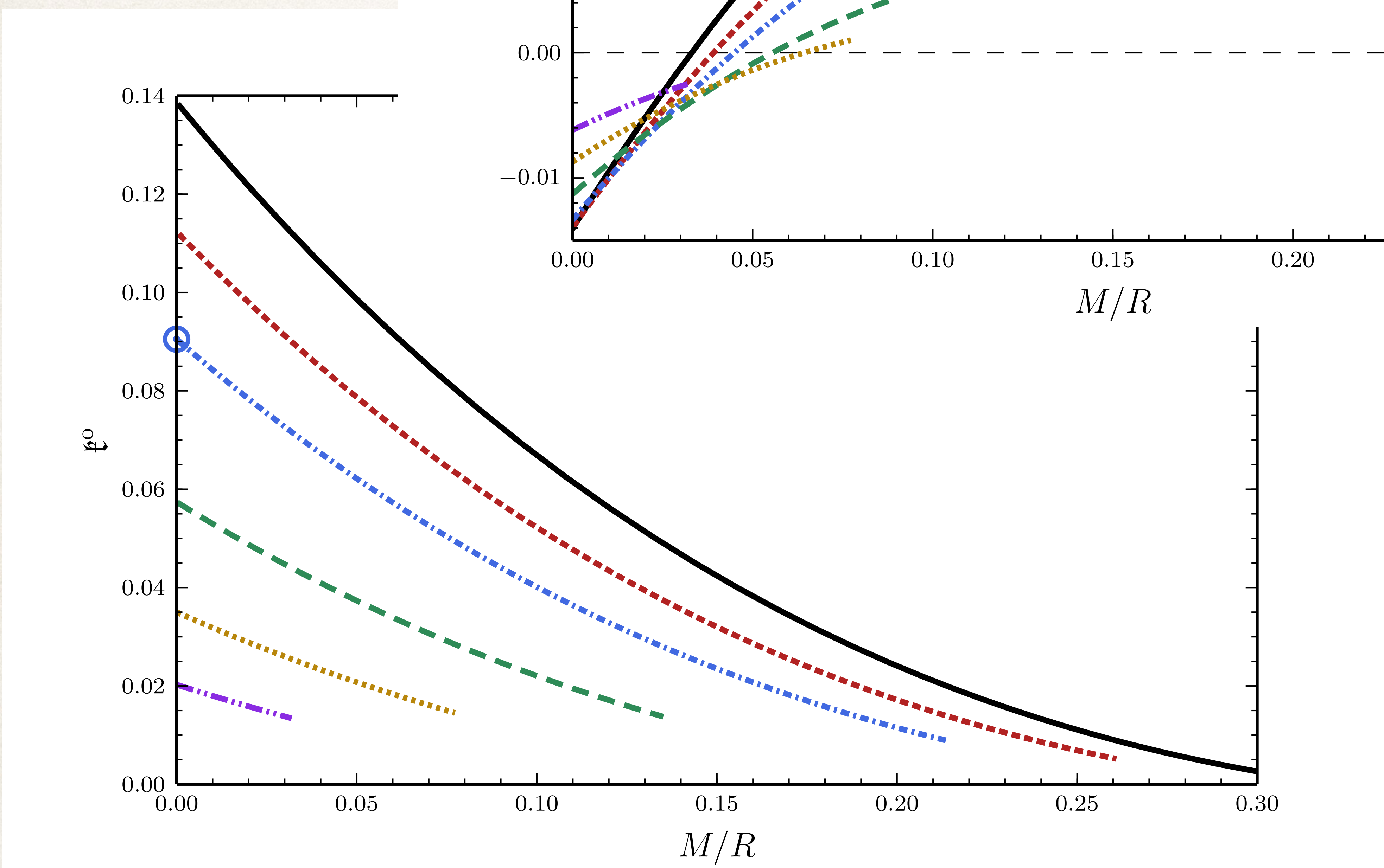
$$\delta g_{tt} \sim \frac{MR^5}{r^4}\, \mathfrak{k}^{\circ}\, \chi_{\langle a}\mathcal{B}_{bc\rangle} n^a n^b n^c$$

$$\delta g_{ta} \sim \frac{M^5 R}{r^4}\, \mathfrak{f}^{\circ}\, \epsilon_{ab}{}^{c}\chi_{\langle c}\mathcal{E}_{de\rangle} n^b n^d n^e$$

$$n^a = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$



$n = 0.5$
$n = 0.75$
$n = 1.0$
$n = 1.5$
$n = 2.0$
$n = 2.5$
0.04
0.03
0.02
0.01
0.00
−0.01
0.00
0.05
0.10
0.15
0.20
0.25
0.30
M/R
$\mathfrak{f}^o$



0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
M/R
$\mathfrak{k}^o$

[Landry (2017)]

Gravitomagnetic tidal currents

- The coupling between the rotational velocity and the gravitomagnetic tidal field produces a force density within the neutron star

$$\boldsymbol{f} = \rho\, \boldsymbol{v} \times \boldsymbol{B}, \qquad \boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{U}$$

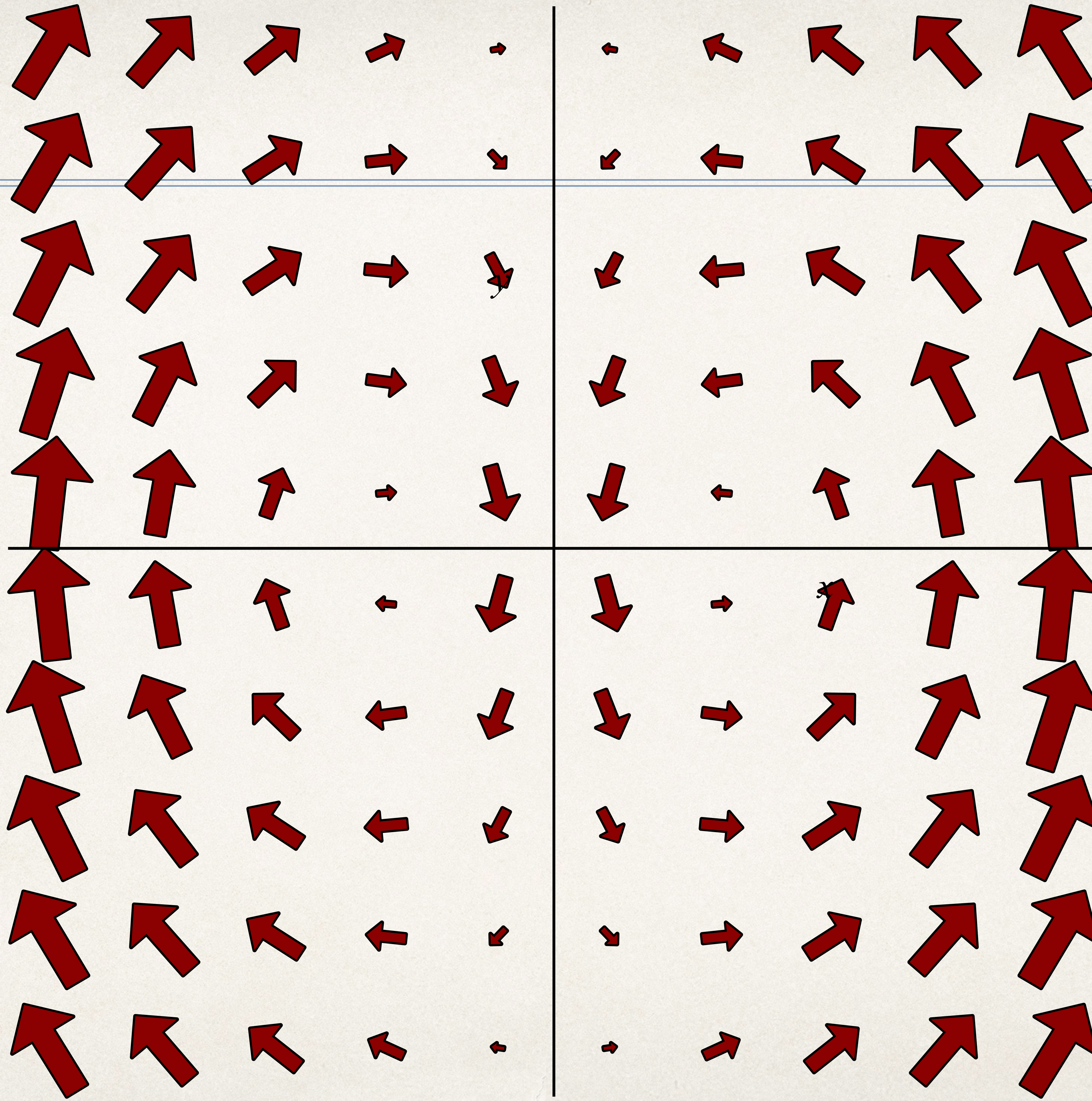
- This force is **not balanced** by pressure gradients

$$\ddot{\xi} + \omega^2 \xi = f$$

- This leads to the development of a **large velocity field**

[Landry and Poisson (2015); Poisson and Doucot (2017); Landry (2017)]

$$\delta v = 2\left(\frac{M'}{1.4\ M_\odot}\right)\left(\frac{2.8\ M_\odot}{M+M'}\right)^{2/3}\left(\frac{R}{12\ \mathrm{km}}\right)^{2}\left(\frac{100\ \mathrm{ms}}{P}\right)\left(\frac{f}{100\ \mathrm{Hz}}\right)^{4/3}\ \mathrm{km/s}$$



y
x

Conclusion

- A compact object undergoes a tidal deformation as it orbits around another compact object
- This affects the orbital motion and the emitted gravitational waves
- This effect can be measured in gravitational waves during the inspiral phase of the binary motion
- A measurement of the tidal Love number reveals information about the internal structure of the compact body
- In the case of a rotating neutron star, the coupling between the rotation and the gravitomagnetic tidal fields generates large tidal currents