The Accelerating Universe
and Other Adventures in Modern Cosmology

Don Salisbury
Austin College
Sherman, Texas

Theorists at Undergraduate Institutions

Kavli Institute for Theoretical Physics - Santa Barbara

Plan of Talk

- Merging exploration, research and pedagogy
- General relativity without tensors
- The accelerating universe
- Cosmic microwave anisotropies
- Text and webpages

Continuing Education

- Opportunities to both enhance foundations and extend beyond specialization
- Limited professional risk
- Undergraduate enhancement at all levels

Curvature without tensors

Curvature of surface of sphere using parallel transport of vectors

Introduce locally flat coordinates $X = R\Phi$ and $Y = R\left(\frac{\pi}{2} - \theta\right)$

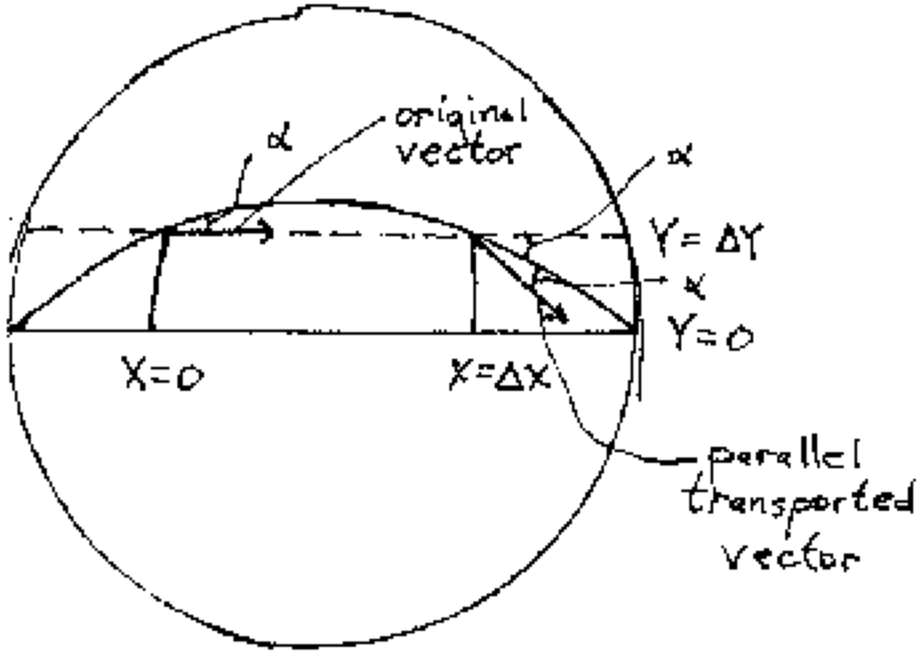
$$ds^2 \approx \left(1 - \frac{Y^2}{R^2}\right) dX^2 + dY^2$$

Geodesic extremizes path length $\int ds$

Parallel vectors in flat space have fixed angle with respect to geodesic - extend rule to curved space

Parallel transport vector by calculating geodesic and then maintaining angle relative to geodesic

$$2\alpha = 2\frac{dY}{dX}\bigg|_{X=\Delta X} = \frac{\Delta X \Delta Y}{R^2} = \Delta\theta\Delta\Phi$$



Einstein's geometrical insight: gravitational trajectories are spacetime geodesics

Analyze gravity in freely falling elevator - a locally flat spacetime coordinate system

ds/c is proper time elapsed on clock carried along trajectory

$$ds^2 = \left(1 - \frac{2GMZ^2}{c^2 r_0^3}\right) c^2 dt^2 - dZ^2$$

2 times Newtonian gravitational potential

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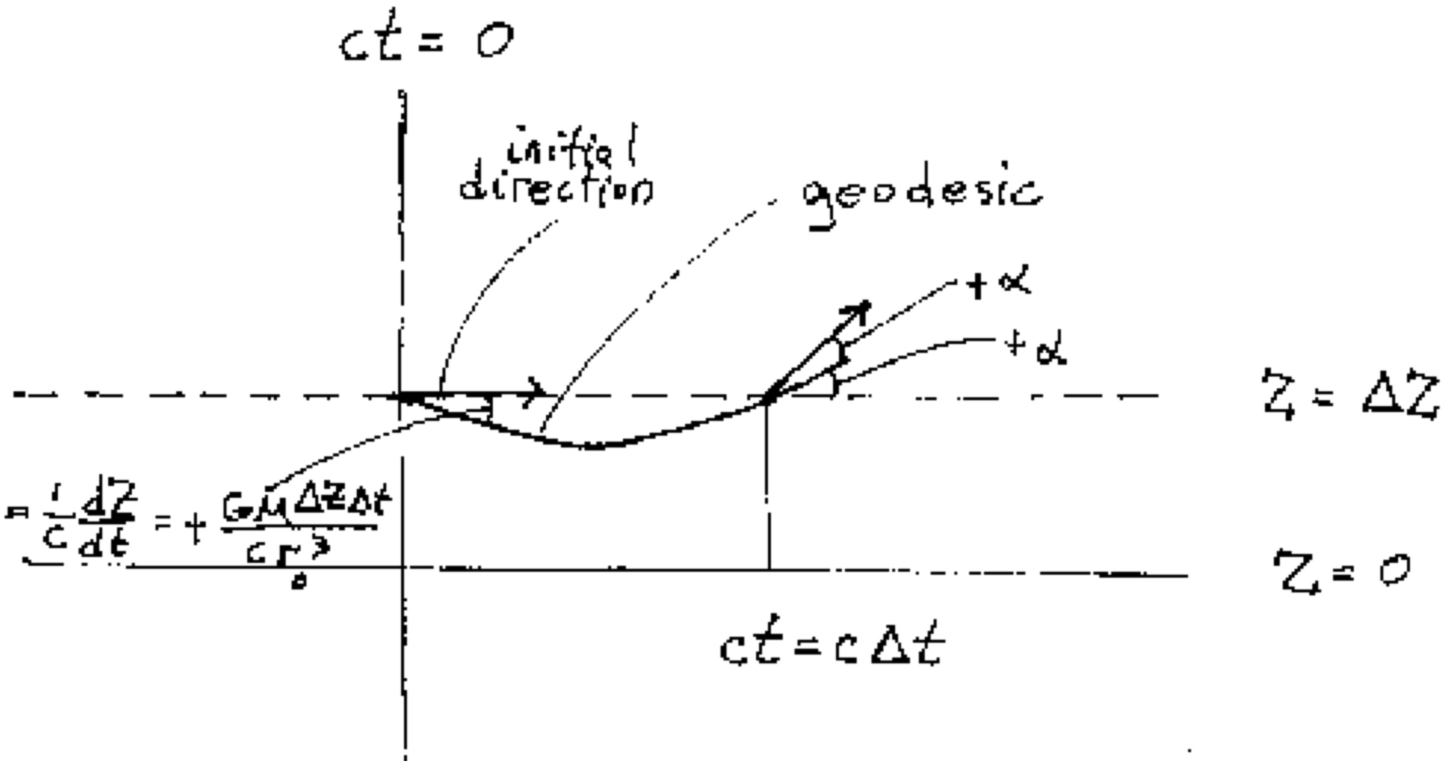
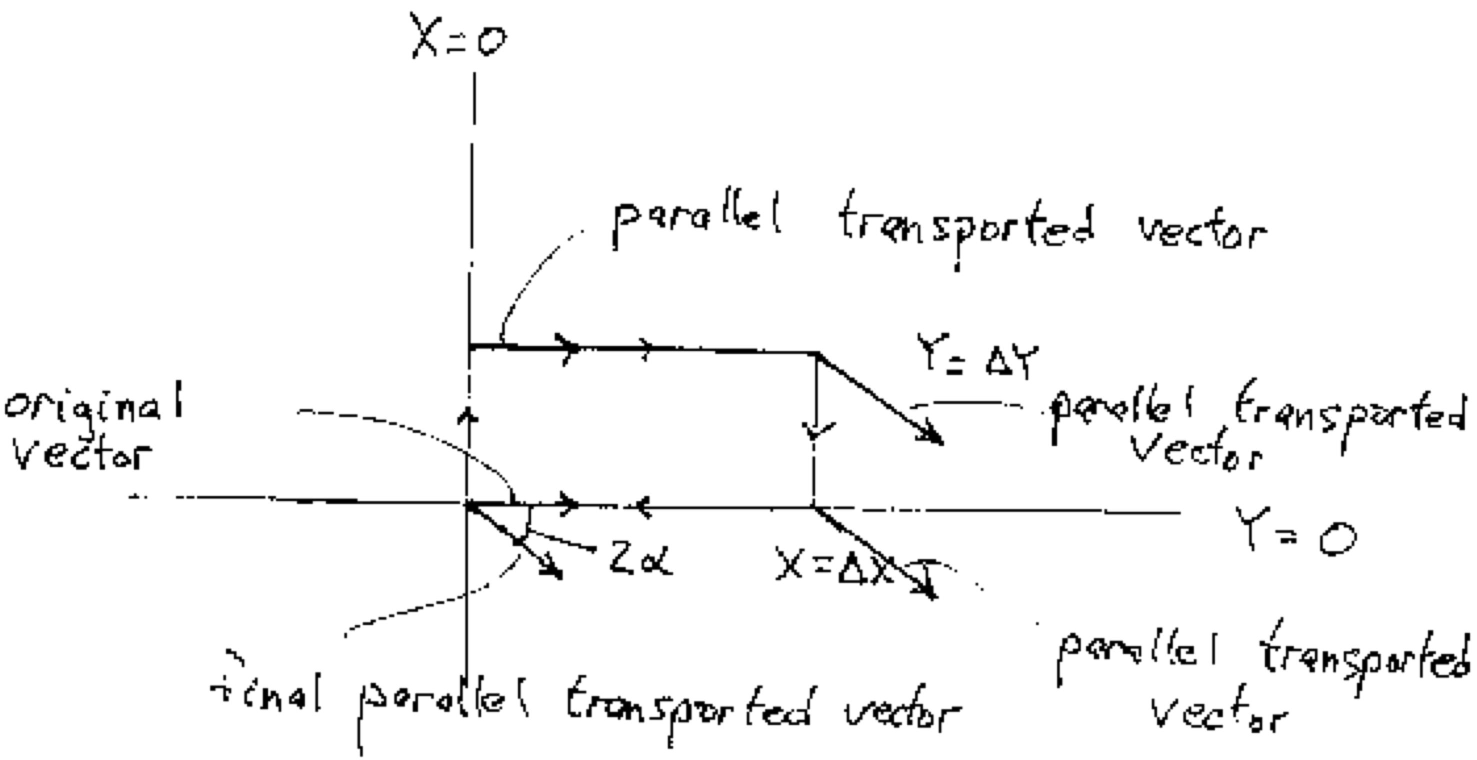
Parallel transport by maintaining angle relative to geodesic - spacetime path of mass tossed downward at $Z = \Delta Z$



$$-\alpha = \frac{1}{c}\frac{dZ}{dt} = +\frac{GM\Delta Z\Delta t}{c r_0^3}$$

Definition of curvature: parallel transport around closed path and divide discrepancy angle by area enclosed by path

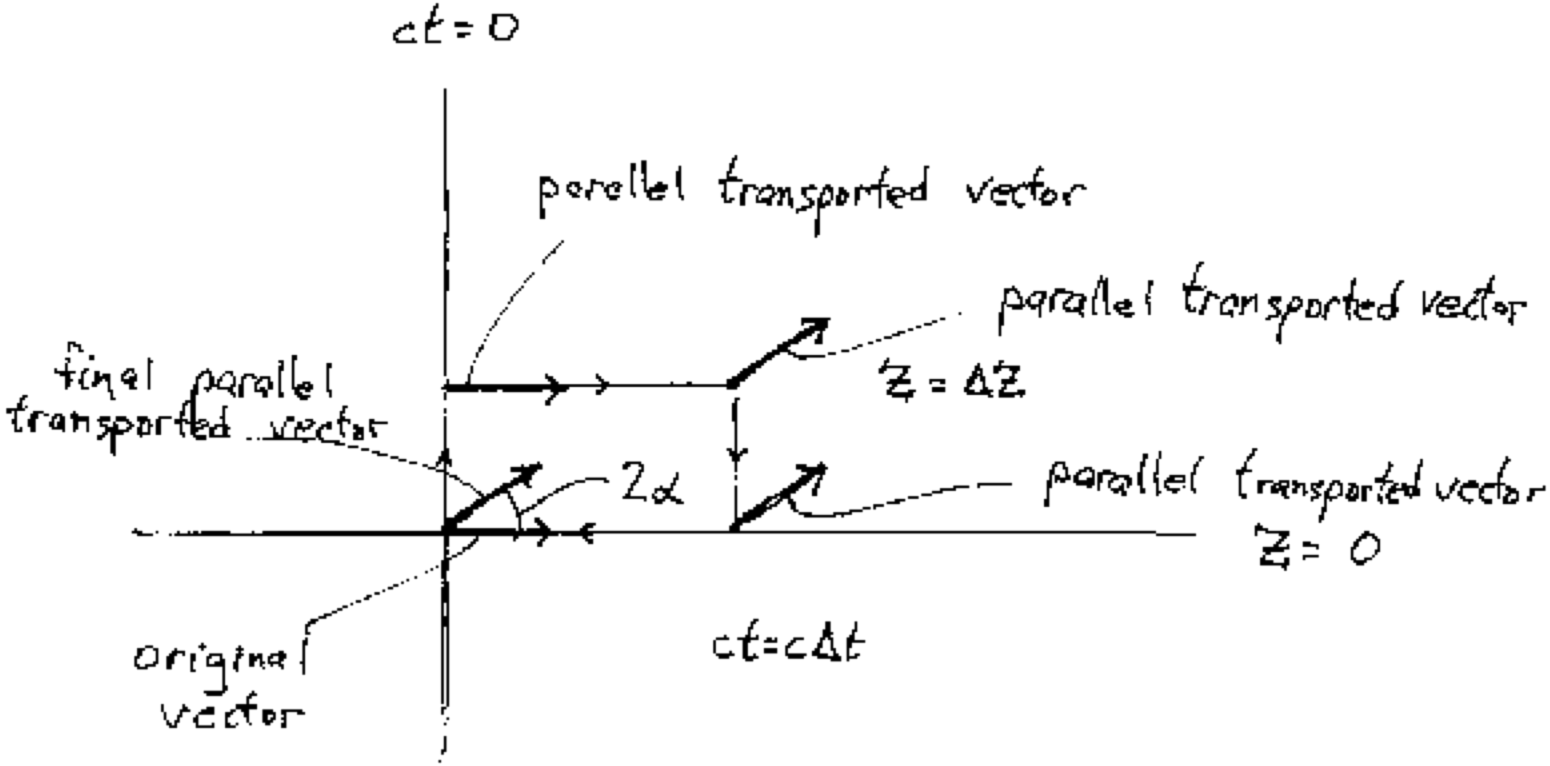
Surface of 2-sphere: $$\frac{\Delta\theta\Delta\Phi}{R^2\Delta\theta\Delta\Phi} = \frac{1}{R^2}$$



Austin College 1849

Free falling elevator:

$$\frac{2GM\Delta Z\Delta t / c r_0^3}{\Delta Z c\Delta t} = \frac{2GM}{c^2 r_0^3}$$



Lessons

- No global inertial frame available when gravitation is present
- Newtonian gravitational potential evident in locally flat inertial coordinates
- Curvature can “almost” be read off line element in locally flat coordinates

Implications for cosmology

Curved 3-space of constant positive curvature

$$ds^2 = dx^2 + dy^2 + dz^2 + \frac{r^2}{R^2 - r^2} dr^2 = \frac{dr^2}{1 - \frac{r^2}{R^2}} + r^2\left(d\theta^2 + \sin^2\theta \;\; d\phi^2\right)$$

FRW line element

$$ds^2 = c^2 dt^2 - a(t)^2\left(\frac{dr^2}{1 - kr^2} + r^2\left(d\theta^2 + \sin^2\theta \;\; d\phi^2\right)\right)$$

Find local inertial coordinates at $t = t_0$ and $r = r_0$ using Newtonian distance $R(t) = a(t)\, r$

$$ds^2 \approx \left(1 - \frac{\ddot{a}_0}{c^2 a_0} R^2\right) c^2 dT^2 - \left(1 + \left(\frac{\dot{a}_0^2}{c^2 a_0^2} - \frac{k}{a_0^2}\right) R^2\right) dR^2 + R^2\left(d\theta^2 + \sin^2\theta \; d\phi^2\right)$$

Newtonian potential

Measures of curvature

Use Newtonian potential to get correspondence limit of Einstein equations with cosmological constant

Recognize curvature terms in Friedmann equation

Friedmann equation

$$\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = \frac{8\pi G}{3c^2}\rho + \frac{\Lambda c^2}{3}$$

curvature

source

Cosmological parameter definitions

Hubble "constant" $H = \frac{\dot{a}}{a}$

critical density $\rho_c = \frac{3c^2 H^2}{8\pi G}$

density parameters $\Omega_i = \frac{\rho_{0\ i}}{\rho_{0\ c}}$

The accelerating universe

Find relation between between observed flux
of distant type Ia supernovae and redshift

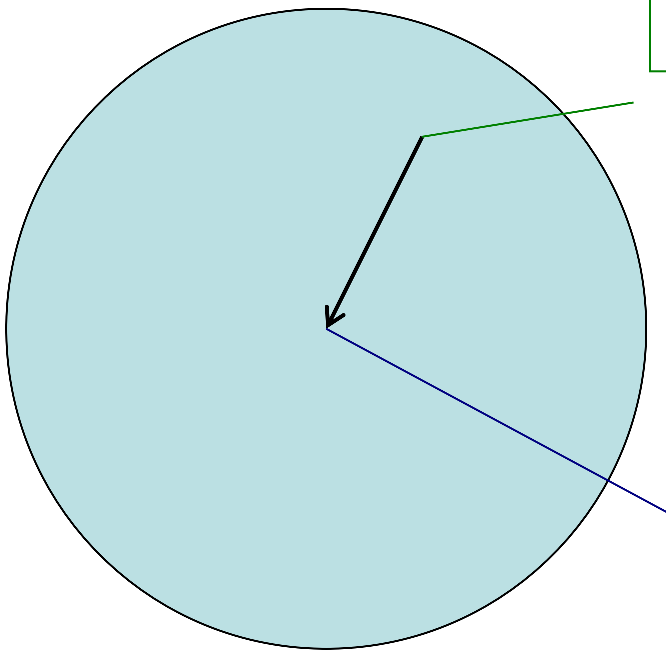
redshift: $1+z = a_0/a$

Trace light on fixed coordinate spatial background
(comoving coordinates)

$$\int \frac{dr}{\sqrt{1-kr^2}} = \int \frac{dt}{a(t)}$$

Change independent variable from t to z to find r(z)

Geometry varies on fixed r-θ (comoving) coordinate background



light leaves SN at coordinate position *r(z)* at time *t(z)*

light arrives at observer at $r = 0$ at time t_0 $(z = 0)$

Spatially flat case - *k = 0*

Friedmann equation:

$$H^2 = H_0^2\left(\frac{\rho_M}{\rho_{0\,M}} + \frac{\rho_\Lambda}{\rho_{0\,\Lambda}}\right) = H_0^2\left(\Omega_M (1+z)^3 + \Omega_\Lambda\right)$$

so

$$r(z) = \int_t^{t_0} \frac{dt}{a(t)} = \frac{1}{a_0}\int_0^z \frac{dz'}{H(z')} = \frac{1}{a_0 H_0}\int_0^z \frac{dz'}{\sqrt{\Omega_M (1+z')^3 + \Omega_\Lambda}}$$

Flux-luminosity relation - in terms of flux and z observed today

$$\text{flux} = \frac{L}{4\pi r(z)^2 a_0^2 (1+z)^2}$$

observed

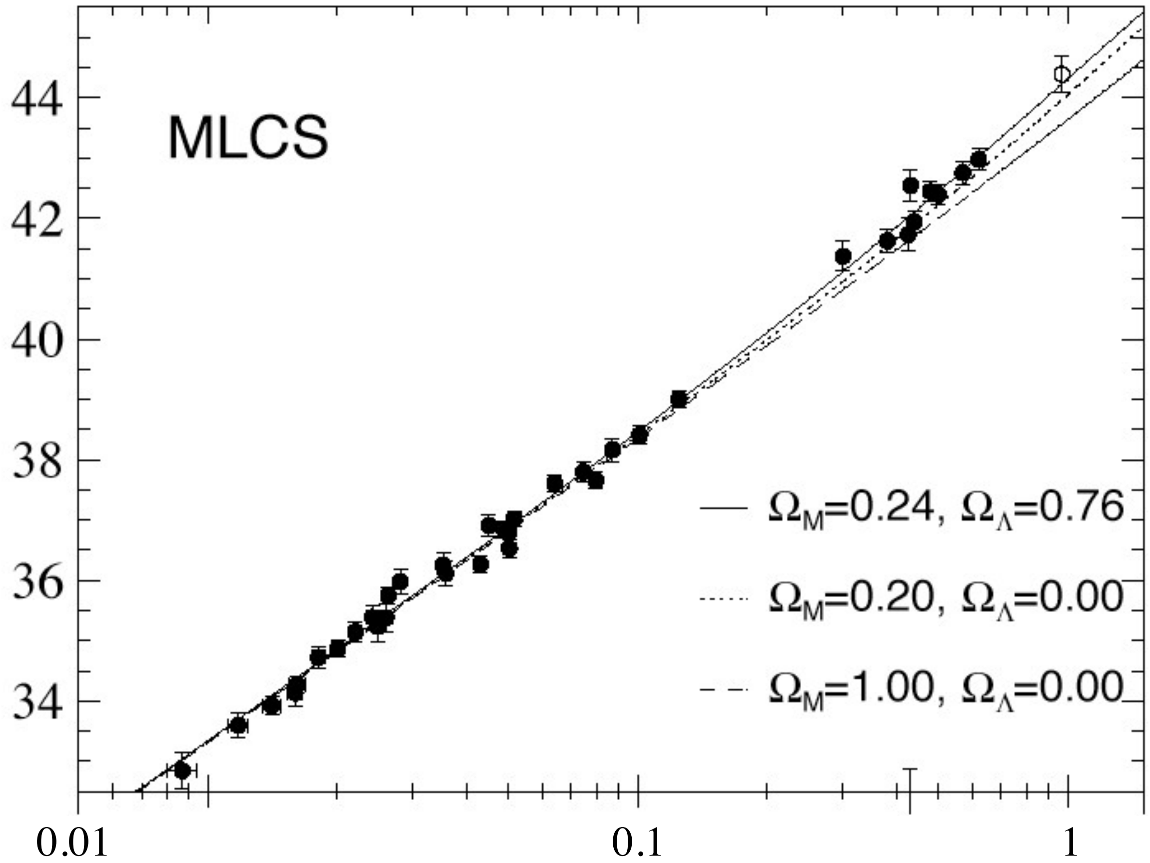
Factor due to time dilation of emission rate and redshift of photons

Best fit determines $\Omega_m = 0.33$ and $\Omega_\Lambda = 0.67$

Austin College honors thesis student Leslie Woerner worked in Brian Schmidt's group at Mt. Stromlo Observatory in January term on modeling SN Ia light curves. Cosmological background learned at AC

Fit from High-Z Supernova Search, Schmidt et. al. *A.J.* ***507*** *(1998)*

apparent minus absolute magnitude



z

Anisotropies in the cosmic microwave background

Junior-senior level special topics course

All AC physics courses require semester project with written paper and public oral presentation

Objectives:

- public tutorial for peers on frontiers in microwave anisotropies
- eventual web tutorial

Acoustic waves

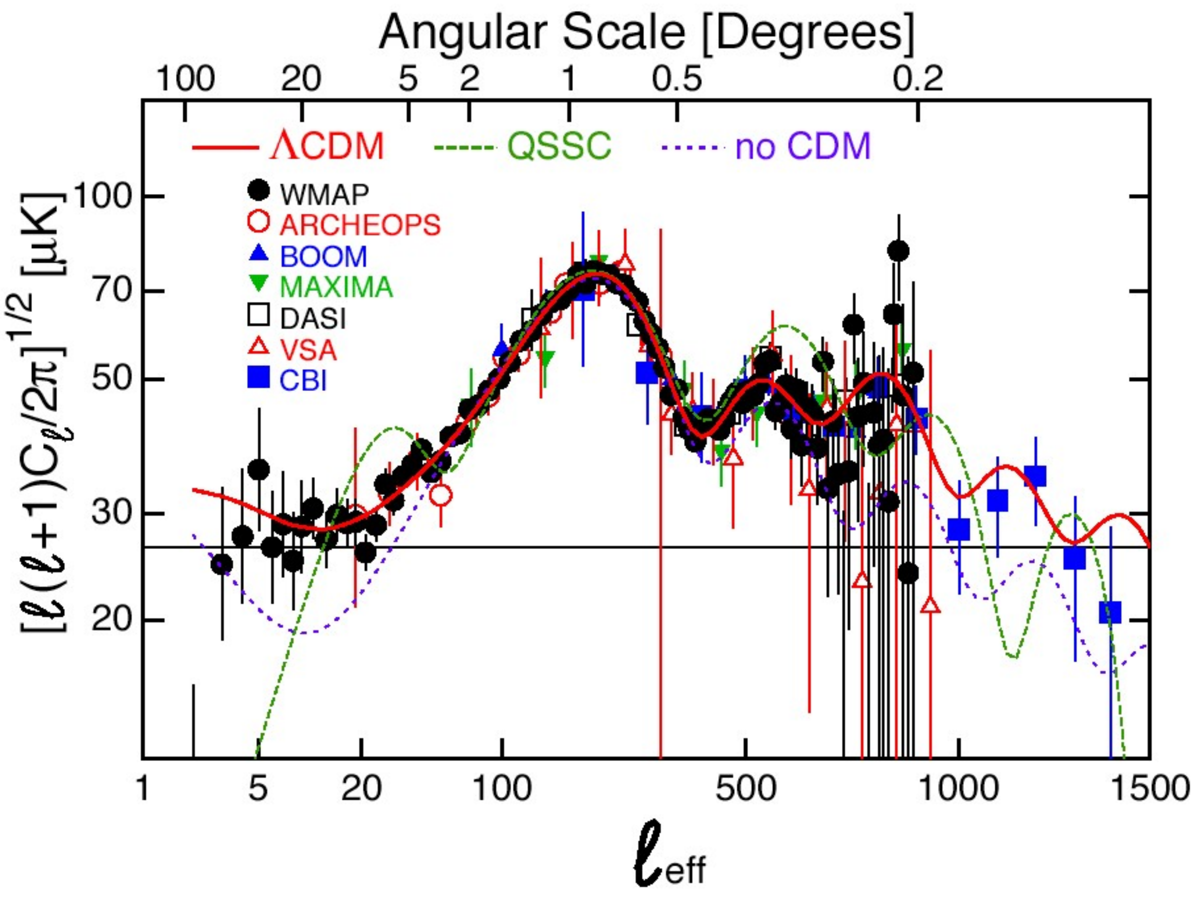
Fixed comoving wavelength mode of oscillation of temperature variation was “derived” starting in Newtonian local inertial coordinates

Students solved resulting variable mass damped harmonic oscillator numerically with variable cosmological parameters, obtaining amplitudes at $z_{decoupling} = 1100$

Results were plotted for random mode directions to produce predicted anisotropy sky map

Temperature deviation power spectrum including WMAP from 2003

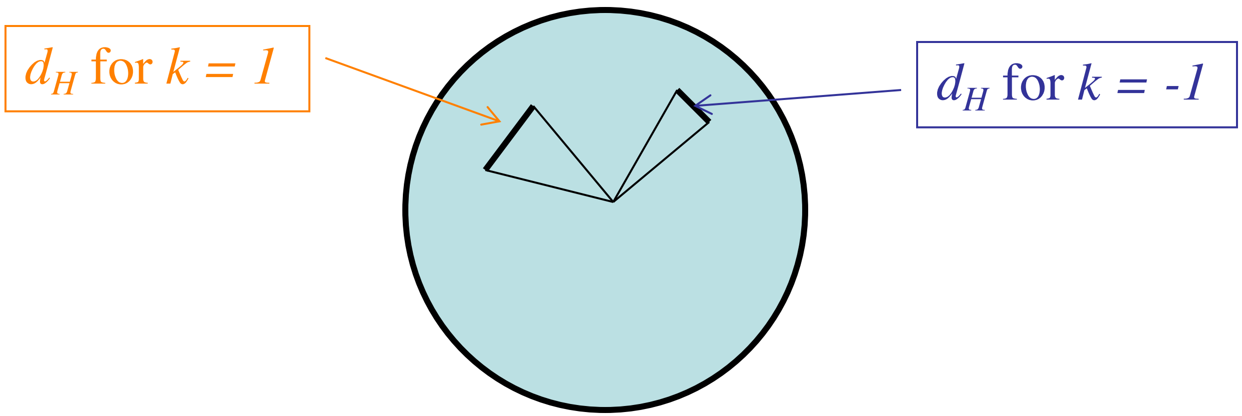
From "The WMAP Data and Results," E. L. Wright, astro-phy/0603132



Curvature dependence of first peak in power spectrum

Largest wavelength in power spectrum is roughly size of horizon at $z = 1100$, $d_H \approx 1/H(z=1100)$

For flat space this corresponds to an angular size of about one degree - angle is smaller for negative spatial curvature and larger for positive spatial curvature (since circumference of a circle is greater than Euclidean value for negative curvature and vice versa)



Current work

Second year introductory cosmology textbook with particle physics collaborators Doris Rosenbaum and Vigdor Teplitz

Collaborative student assisted creation of cosmology tutorial web pages with java simulations