The Passage of Time in Einstein's Universe

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UTEP Joint Physics/Philosophy Seminar
November 17, 2006

Plan of Talk

1. Pre-Einstein concepts of time
2. Time in Einstein's Special Theory of Relativity
3. Implications of global symmetry
4. Local symmetry and the initial value problem
5. Singular Lagrangian prehistory and Leon Rosenfeld
6. General coordinate symmetry in the Hamiltonian formulation of general relativity
7. Relative ontological time in Einstein's universe?
8. Implications for quantum gravity

I - Time before Einstein

- Aristotle’s plenum and the void
- Descarte’s relational mechanics
- Newton’s time

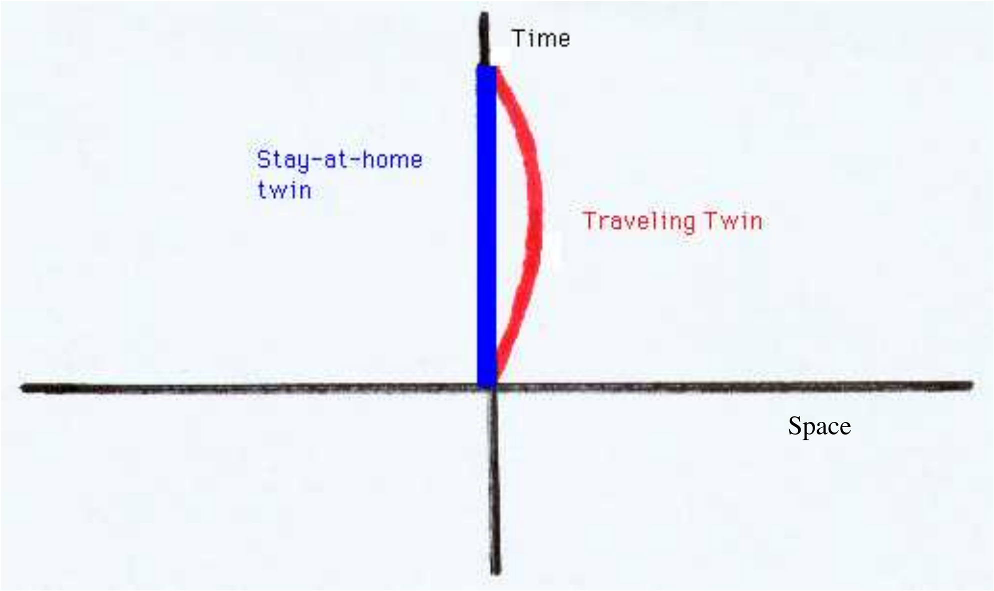
I. Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external, and by another name is called duration: relative, apparent, and common time, is some sensible and external (whether accurate or unequable) measure of duration by the means of motion, which is commonly used instead of true time; such as an hour, a day, a month, a year.

…Absolute time, in astronomy, is distinguished from relative, by the equation or correction of the apparent time…The duration or perseverance of the existence of things remains the same, whether the motions are swift or slow, or none at all: and therefore this duration ought to be distinguished from what are only sensible measures thereof; and from which we deduce it, by means of the astronomical equation. The necessity of this equation, for determining the times of a phenomenon, is evinced as well from the experiments of the pendulum clock, as by eclipses of the satellites of Jupiter.

II - Time in Einstein's Special Theory of Relativity

- Observers moving at constant velocity relative to each other agree on laws of motion
- Consequence: elapsed time depends on who is measuring it!
  - Primacy of "personal time"
  - Example: spacetime diagram of traveling and stay-at-home twin

Flat spacetime geometry



Red twin ages less than blue twin

Einstein's interpretation: "spacetime distance" along red path is less than along blue path

"Straightest" spacetime path is the longest!

III - Global symmetry

- Asymmetry transformation does not change the form of dynamical equations
- Example: Dynamical equation of free fall where *z* is the height, *t* the time, and $g = 9.8\ \mathrm{m/s^2}$

$$\frac{d^2 z}{dt^2} = g$$

A particular solution: $z = -\frac{1}{2} g t^2$

- Redefine time zero: $t' = t + 2$
- Then the form of the dynamical equation is unchanged:

$$\frac{d^2 z}{dt'^2} = g$$

- Consequence: substitution into original solution yields a new solution:

$$z = -\frac{1}{2} g (t+2)^2 = -2g + 2gt - \frac{1}{2} g t^2$$

- This is instance of Emmy Noether's first theorem (1918)

IV - Local symmetry and initial value problem

- Free electromagnetic field example. V is the electrostatic potential and A_x, A_y, and A_z the vector potential.

- The dynamical equations do not change their form under the redefinitions of the fields, where λ is an arbitrary function

$$V' = V + \frac{\partial \lambda(x,y,z,t)}{\partial t}, \qquad A'_x = A_x + \frac{\partial \lambda(x,y,z,t)}{\partial x}, \qquad \text{etc}$$

- Implication of Noether's second theorem: solutions of dynamical equations contain an arbitrary function, so solutions are not uniquely determined by initial conditions

- But there is no loss of physical determinancy since only the electric and magnetic fields are physically observable. For example, the electric field is

$$E_x = -\frac{\partial V}{\partial x} + \frac{\partial A_x}{\partial t}$$

V - Leon Rosenfeld and his pioneering work

- Born Belgium 1904
- Doctorate Liege 1926
- Research in Paris, Göttingen, Zurich 1926-1930
- Taught theoretical physics at Liege, Utrecht, Manchester, Copenhagen 1940-1974
- Collaborators and correspondents: Bohr, Pauli, de Broglie, Dirac, Heisenberg, Infeld, Klein …
- Died October 1974

Heisenberg and Rosenfeld

Rosenfeld (right, standing)
at 1933 Solvay Meeting

The initial value (Hamiltonian) formulation of electromagnetism

- Required to take canonical route to quantization of the elctromagnetic field
- Must rewrite equations of motion so as to contain only first derivatives with respect to time. This is done by defining new variables, the “momenta”, in terms of the velocities.

  For example, in the case of the object in free fall, let $$p = \frac{dz}{dt}$$

  then this definition plus the equation of motion $$\frac{dp}{dt} = -g$$

  become the Hamiltonian equations of motion. These equations of motion are determined by a “Hamiltonian”

$$H = gz + \frac{1}{2}p^2$$

- But there is a problem with electromagnetism. One of the canonical momenta (the one associated with the time derivative of the electrostatic potential) vanishes identically!

Proposals for dealing with vanishing momentum:

Heisenberg/Pauli formalism (1929-1930)

- Add non-gauge invariant term to Lagrangian (destroys local symmetry)
- Or set $V = constant$ (destroys manifest Lorentz invariance)

Pauli: "Ich warne Neugierige"

Rosenfeld's debt to Pauli: "*As I was investigating these relations in the especially instructive example of gravitation theory, Professor Pauli helpfully indicated to me the principles of a simpler and more natural manner of applying the Hamiltionian procedure in the presence of identities*" (My translation from Rosenfeld's 1930 paper)

Rosenfeld's formal constraint analysis in "On the quantization of wave fields", Annalen der Physik 1930

Zur Quantelung der Wellenfelder

Von L. Rosenfeld

Einleitung

Wesentliche Fortschritte in der Formulierung der allgemeinen Quantengesetze der elektromagnetischen und materiellen Wellenfelder haben neuerdings Heisenberg und Pauli[1]) erzielt, indem sie die von Dirac erfundene „Methode der nochmaligen Quantelung" systematisch entwickelten. Neben gewissen sachlichen Schwierigkeiten, die viel tiefer liegen, trat dabei eine eigentümliche Schwierigkeit formaler Natur auf: der zum skalaren Potential kanonisch konjugierte Impuls verschwindet identisch, so daß die Aufstellung der Hamiltonschen Funktion und der Vertauschungsrelationen nicht ohne weiteres gelingt. Zur Beseitigung dieser Schwierigkeit sind bisher drei Methoden vorgeschlagen worden, die zwar ihren Zweck erfüllen, aber doch schwerlich als befriedigend betrachtet werden können.

1. Die erste Heisenberg-Paulische Methode ist ein rein analytischer Kunstgriff.[2]) Man fügt zur Lagrangefunktion gewisse Zusatzglieder hinzu, die mit einem kleinen Parameter ε multipliziert sind und bewirken, daß der obenerwähnte Impuls nicht mehr verschwindet. In den Schlußresultaten muß man dann zum Limes $\varepsilon = 0$ übergehen. Die ε-Glieder führen aber zu unphysikalischen Rechenkomplikationen[3]) und zerstören die charakteristische Invarianz der Lagrangefunktion gegenüber der Eichinvarianzgruppe.

2. Die zweite Heisenberg-Paulische Methode[4]) benutzt hingegen wesentlich diese Invarianz. Dem skalaren Potential

1) W. Heisenberg u. W. Pauli, Ztschr. f. Phys. **56**. S. 1. 1929; ebenda **59**. S. 168. 1930. Im folgenden mit H. P. I bzw. II zitiert.
2) H. P. I, S. 24—26, 30ff.
3) Vgl. L. Rosenfeld, Ztschr. f. Phys. **58**. S. 540. 1929.
4) H. P. II.

Rosenfeld's formal constraint analysis in "On the quantization of wave fields", Annalen der Physik 1930

- Local symmetries always lead to
  - non-unique evolution in time
  - constraining relations among variables and associated momenta
  - Hamiltonian (from which equations of motion are determined) constructed using the constraints
  - vanishing of Hamiltonian if, as in general relativity, the equations of motion take the same form for arbitrary choices of the time coordinate
- Rosenfeld was first to consider how to implement local symmetry-induced transformations on Hamiltonian variables
- Rosenfeld's dynamical model - gravitation with a charged spinorial

Dirac field source

Origins of the model

- Weyl/Fock coupling of Dirac field with gravity - 1929
- Tetrads and Weyl's reinterpretation of gauge symmetry
  See analyses by Scholz (physics/0409158) and Straumann (hep-ph/0509116)

VI -The symmetry under general coordinate transformations of general relativity

- There is no preferred way of assigning spatial or temporal coordinates in general relativity
- But - we do now have a way - following the pioneering work of Rosenfeld, Bergmann, and Dirac - of tracking the evolution from an initial instant for any choice we wish to make for a temporal coordinate
- Bergmann and Komar (1972), following up on the work of Paul Dirac (1958), made the first step in understanding how general coordinate symmetry is preserved in the initial value (Hamiltonian) version of general relativity
- Pons, Salisbury and Shepley (1997-2001) showed that the underlying initial value (Hamiltonian) symmetry is relational in the sense that the symmetries depend not only on arbitrary spacetime functions - but necessarily also on the physical gravitational field.
- Pons and Salisbury (2005) explained how to construct a univocal relational time, exploiting the newly discovered Hamiltonian symmetry. An “intrinsic” time is defined using an appropriate function of physical fields.

Brief Bergmann biography

- Born Berlin-Charlottenburg 1915
- Mother Dr. Emmy Bergmann moved with children to Freiburg 1922 - she and sister emigrated to Israel 1935
- Father Dr. Max Bergmann 1921 - 1933 head of Institut für Lederforschung, Dresden (now Max Bergmann Zentrum für Biomaterialen)
- Prague, Charles University degree 1936
- Einstein Assistant 1936 - 1941: unified field theory
- Syracuse University 1947 - 1982
- Died October 2002

Excerpt of letter from Peter Bergmann to Nathan Rosen, dated September 26, 1973

Dirac is perhaps the last of the really great pioneers that created today's physics. Though he may not be able to last through a heavy conference schedule, he will take in a few papers a day every day for a week, and he will make very helpful and acute comments on occasion. His presence will, of course, lend prestige to GRG7, but he will be a real asset as a physicist. Having through an extended period wrestled with the same problems that he succeeded in solving - a viable Hamiltonian version of general relativity, I have the profoundest respect for his genius, second only (in my personal experience) to Einstein. I think that you should act soon.

VI - A cosmological example of relative ontological(?) time

- Isotropic expanding universe containing a massless scalar field, with two gravitational (metric) variables, the spatial metric (expansion factor) a and the lapse function N
- The Hamiltonian model is symmetric under the small time transformation

$$t' = t - \frac{\xi(t)}{N(t)}$$

- Choose the square of the expansion factor as the intrinsic time since it increases monotonically with coordinate time
- The model fixes a unique correlation between the value of a^2 and the value of the scalar field
- It can be shown explicitly that the resulting fields are invariant under the group of transformations given above - thus we have true evolution in intrinsic time, but only when there is stuff in the universe!

VII - Implications for quantum gravity

- In the loop approach to quantum gravity a^2 can take only certain discrete values, determined in terms of the Planck time (about 10^{-43} seconds)
- Although most researchers in the field are satisfied that no notion of temporal evolution need be present in the Planckian era, I maintain that one can sensibly construct a generalized Schroedinger quantum time stepping.

- Most of the quantum relativity community is still convinced that quantum time is “frozen”, yet most also recognize the possibility of non-trivial evolution in intrinsic time.
- There is much work to be done!