Leon Rosenfeld and the challenge of the vanishing momentum in quantum electrodynamics

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HQ2 - Utrecht - July 17, 2008

Plan of Talk

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2. Quantum electrodynamics to 1930
3. Leon Rosenfeld background
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5. Rosenfeld constrained Hamiltonian dynamics formalism
6. Dirac and Rosenfeld
7. The impact of Rosenfeld's work

2. Quantum electrodynamics to 1930

- Dreimännerarbeit (1926)
- Dirac quantization of electromagnetic field (1927)
- Jordan quantum mechanics of a Fermi gas (1927)
- Jordan and Klein second quantization of massive bosons (1927)
- Jordan and Pauli relativistic second quantization of free electromagnetic field (1928)
- Jordan and Wigner field anticommutation relations (1928)
- Dirac relativistic wave equation (1928)
- Pauli and Heisenberg second quantization of interacting electromagnetic and electron fields (1929) and (1930)

Pauli, Jordan, Heisenberg and second quantization

- Heisenberg agrees Pauli quantization program (H to P, 2/23/27) References in letters from Pauli to Jordan (3/12/27) and to Wentzel (5/16/27) to study of functional techniques

Aber hierbei sind noch ungelöste Schwierigkeiten geblieben und ich habe noch nichts Fertiges darüber. (But there are still unsolved difficulties and I don't yet have anything definitive to say about it.)

- Pauli to Heisenberg (5/16/27)

Uber die Hamiltonsche Wechselwirkung zwishen Strahlung und Materie hab' ich noch keine entgültige Meinung, ich muss Dirac erst studieren. (I don't yet have a final opinion on the Hamiltonian interaction between radiation and matter. I must study Dirac.)

- Pauli to Wentzel (6/23/27)

Hinsichtlich der Möglichkeit, mit der Quantenelectrodynamik weiterzukommen, bin ich wieder sehr skeptisch. (Concerning the possibility of proceeding further with quantum electrodynamics I am once again very skeptical.)

- Pauli to Bohr (8/6/27)

Jordan und mir ist es gelungen, diese Bedingungen so zu formulieren, dass nicht mehr, wie bei Dirac, die Raum-Koordinaten vor der Zeit ausgezeichnet, sondern vielmehr alle Forderungen der relativistischen Invarianz erfüllt sind.
(Jordan and I have succeeded in formulating these conditions so that, as opposed to Dirac where the position coordinates are distinguished from the time, all of the demands of relativistic invariance are fulfilled.)

Die blosse Elektrodynamik des ladungsfreien Raumes kann natürlich noch keine Messungsmöglichkeiten der Feldstärken wirklich angeben. Dies wird erst durch Betrachtungen über die Elektrodynamik mit Teilchen möglich sein. Aber immerhin wird für diese doch bereits ein allgemeiner Rahmen geschaffen, in das sie hineinpasen muss. Zum Nachdenken über die relativistische Behandlung des Mehrkörpersproblems werde ich aber erst nach Ferien kommen, vorläufig wissen weder Jordan noch ich etwas bestimmtes darüber. Immerhin habe ich doch gewisse Hoffnungen.
(The pure electrodynamics of the charge-free space can naturally not offer the possibility of measuring field strengths. This will only be possible with electrodynamic observations with particles. But nevertheless we have created a general framework that will be appropriate for their inclusion. I will not be able to think about the relativistic treatment of the many-body problem until after the vacation. Meanwhile neither Jordan or I know anything definite about it. Nevertheless I have some hope.)

- Pauli to Kramers (2/7/28)

Inzwischen haben Heisenberg und ich gemeinsam auch den Fall des Vorhandensein geladener Teilchen in analoger (relativistische-invarianter) Weise zu behandeln versucht, wobei wir uns auch wesenlich auf die Resultate von Jordan und Klein stützen. Es scheint in der Tat zu gehen, aber es ist noch nicht alles fertig; wir haben Schwierigkeiten and der Stelle, die der Umstellung der Faktoren im Energieausdruck (Elimination der Wechselwirkungsenergie der Teilchen mit sich selber) bei Klein und Jordan entspricht.

(In the meantime Heisenberg and I have together tried to treat the presence of charged particles in an analogous (relativistically invariant) manner. We rely essentially on the results of Jordan and Klein. It seems to work, but it's not yet finished; we have difficulties at that point corresponding to Klein and Jordan's reordering of factors in the energy expression (elimination of the self-energy of the particle).

- Pauli to Dirac (2/17/28)

…möchte ich Sie um Ihre Meinung fragen betreffend eine wesentliche physikalische Schwierigkeit, die in dem System von Heisenberg und mir auftritt und die wir nicht beseitegen konnten. (I would like to ask your opinion concerning an essential physical difficulty that has come up in the model of Heisenberg and myself. We have been unable to resolve it.)

- Pauli to Bohr (6/16/28)

In der Quantenelektrodynamik bin ich gar nicht mehr vorwärts gekommen (Heisenberg übrigens auch nicht). Die Schwierigkeiten von denen ich bei meinem Besuch in Kopengagen erzählte, scheinen doch von sehr tiefliegender Art zu sein und ich glaube jetzt, dass sie erst durch eine prinzipiell neue Idee umgangen werden können. (War Dirac in Kopenhagen? Was meint er zur jetztigen Situation?)
(I have hardly made any progress with quantum electrodynamics. Neither has Heisenberg. The difficulties that I described to you in my visit to Copenhagen seem to be very deep and I think that they will be overcome only with a basically new idea. (Was Dirac in Copenhagen? What does he think about this current situation?))

- Pauli to Bohr (1/16/29)

Mit meinen eigenen Arbeiten geht es leider vorläufig gar nicht ... Darüber schämte ich mich so, dass ich es immer hinaus- geschoben habe, Dir zu schreiben. ... Ich bin nur dumm und faul… Heisenberg hat eine Idee, wie unser Ansatz zum relativistischen Mehrkörperproblem vielleicht doch durchgeführt werden könnte. (Unfortunately nothing is happening with my own work … I have been so ashamed about this that I have been putting off writing to you…I'm just stupid and lazy… Heisenberg has an idea how our relativistic many-body model can perhaps be worked out.)

- Pauli to O. Klein (2/18/29)

Zu meinem eigenen Amusement machte ich damals einen kurzen Entwurf zu einem utopischen Roman, der den Titel Gullivers Reise nach Uranien haben sollte und im Stile vom Swift als politische Satire gegen die heute Demokratie gedacht war, nämlich gegen alles, was auch nur entfernt nach Parlamenten, Abstimmungen und Majoritäten riecht! In Solchen Träumen befangen, kam mir im Januar plötzlich eine Nachricht von Heisenberg zu, dass er mittels eines (im Folgenden näher zu erläutern) Kunstgriffes die formalen Schwierigkeiten beseitigen kann, die der Durchführung unserer Quantenelektrodynamik entgegen standen, so dass das relativistische Mehrkörperproblem jetzt gewissermassen gelöst ist!

(For my own amusement I put together at that time a short outline for a utopian novel, with the intended title Gulliver's Travels to Uranien. It was conceived in Swift's style as a political satire on contemporary democracy, against all that only remotely smacks of parlaments, elections and majorities. Caught up in such dreams there came to me suddenly in January news from Heisenberg (to be explained in the following). He had found an artificial device to remove the formal difficulties that had stood in the way of our completion of quantum electrodynamics. Our relativisitic many-body problem is practically solved!)

The vanishing momentum problem

The original choice for the many-body Lagrangian was

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - eA_\mu\bar{\psi}\gamma^\mu\psi + i\hbar c\bar{\psi}\gamma^\mu\psi_{,\mu} - mc^2\bar{\psi}\psi$$

The corresponding momentum conjugate to A_μ is

$$p^\mu = \frac{\partial\mathcal{L}}{\partial\dot{A}_\mu} = F^{0\mu} \text{ so } p^0 \equiv 0$$

But a vanishing p^0 is inconsistent with the commutation relations

$$\left[A_0(x), p^0(x')\right] = i\hbar\delta^3(x - x')$$

Heisenberg's *Kunstgriff* - Pauli and Heisenberg I (1929)

Destroy invariance under gauge transformations $\delta A_\mu = \xi_{,\mu}$ and $\delta\psi = \frac{ie}{\hbar c}\psi\xi$
by adding a gauge breaking term to the Lagrangian $\frac{\epsilon}{2}(A^\mu_{,\mu})^2$

$$\mathcal{L}' = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - eA_\mu\bar{\psi}\gamma^\mu\psi + i\hbar c\bar{\psi}\gamma^\mu\psi_{,\mu} - mc^2\bar{\psi}\psi + \frac{\epsilon}{2}\left(A^\mu_{,\mu}\right)^2$$

Then take the limit $\epsilon \to 0$ at completion of calculations

Remaining problems: infinite zero-point energy and infinite self-energy of the electron in second order perturbation theory

Fermi (1929)

Introduced Fourier decomposition with respect to transverse, longitudinal, and scalar coefficients in the Lorentz gauge.

Pauli and Heisenberg II (1930)

- Set $A_0 = 0$ But then manifest Lorentz invariance is destroyed.

- Derived Fermi theory from the Lagrangian

$$\mathcal{L}' = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - eA_\mu\bar{\psi}\gamma^\mu\psi + i\hbar c\bar{\psi}\gamma^\mu\psi_{,\mu} - mc^2\bar{\psi}\psi + \frac{1}{2}\left(A^\mu_{,\mu}\right)^2$$

Pointed out that Lorentz condition $A^\mu_{,\mu} = 0$ must be understood as condition on initial states.

Heisenberg and Pauli's self-assessments

Also in those papers which Pauli and I wrote on the quantization of fields we saw quite soon that after all it didn't look too well. It is true that for the free light quanta everything could be made to fit, but as soon as interaction came in it didn't look right (Heisenberg, AHQM 2/28/63, p 22)

Here, in electrodynamics, it didn't become simple… For instance, you had to introduce this supplementary condition and you had to make some kind of limiting process -- first introducing an epsilon and at the end you put epsilon to zero. You know, that kind of stuff didn't look right. (Heisenberg, AHQM 2/28/63, p 23)

Already there it was a bit artificial to do the Lorentz condition without introducing the indefinite metric. Well, finally Pauli and I succeeded in replacing it by some symmetry argument, but again it was a bit funny. You could say that the fourth Maxwell equation is not a rigorous operator equation, it's only a supplementary condition to the --, Well, you know. It came into the region of the "Ausrede." (Heisenberg, AHQM 7/28/63, p 7)

3. Leon Rosenfeld background

Born Belgium 1904
Doctorate Liege 1926
Research in Paris under direction of de Broglie, Brillouin, and Lengevin 1926-27
Research in Göttingen under direction of Born 1927-28
Research in Zürich under Pauli 1929-30
Beginning in 1930 several months in Copenhagen in collaboration with Bohr
Taught theoretical physics at Liege 1930-37, Utrecht 1940-47, Manchester 1947-58
NORDITA, Copenhagen 1958-197
Collaborators and correspondents: Bohr, Pauli, de Broglie, Dirac, Heisenberg, Infeld, Klein
…Died October 1974

4. Rosenfeld and Pauli

I came to Zürich before the summer semester… I came from Göttingen where I was still at the time. I had already corresponded with Bohr, asking him whether I could come to Copenhagen… and so I wrote to Pauli then to ask him if he would take me up. He was very friendly and he said: "With pleasure, because we have just completed a scheme of quantum electrodynamics with Heisenberg; 'das ist ein Gebiet, das noch nicht abgebrochen ist.'" So he was eager to have people brush up the details and explore the consequences and that is what I did at Zürich actually (AHQM 7/19/63, p 5)

… I got provoked by Pauli to tackle this problem of the quantization of gravitation and the gravitation effects of light quanta, which at that time were more interesting. When I explained to Pauli what I wanted to work out, I think it was the Kerr effect or some optical effect, he said "Well, you may do that, and I am glad beforehand for any result you may find." That was a way of saying that this was a problem that was not instructive, that any result might come out, whereas at that time, the calculation of the self energy of the light quantum arising from its gravitational field was done with a very definite purpose. (AHQM 7/19/63, p 8)

…Then Pauli told me that he was not at all pleased with longitudinal waves, so he wanted to have them treated another way, which I did, but that was not more enlightening, far from it. (AHQM 7/19/63, p 9)

… There was this point in their proof in which the invariants of the Hamiltonian seemed to depend on a special structure of the Hamiltonian, and that looked suspicious… "Yes, I understand that [said Pauli], but we have not been able to find a mistake in our calculation and we do not understand what this means; we suspect that it must be wrong, but we don't know." Then the thing came to a crisis through the fact that I tried to make a more general formulation of field quantization … It was a purely abstract scheme which worked in a completely general way with only this complication of accessary conditions, but at any rate, not due to any special structure but only to the existence of invariance with respect to a group. So at that stage I was convinced that there must be a mistake in the original paper… (AHQM 7/19/63, p 5)

5. Rosenfeld's formal constraint analysis in "On the quantization of wave fields", Annalen der Physik, 113 - 152 (1930)

Application to quantum electrodynamics addressed in "La théorie quantique des champs", Annales de l'Institut Henri Poincaré, 25 - 91 (1932)

Zur Quantelung der Wellenfelder

Von L. Rosenfeld

Einleitung

Wesentliche Fortschritte in der Formulierung der allgemeinen Quantengesetze der elektromagnetischen und materiellen Wellenfelder haben neuerdings Heisenberg und Pauli[1]) erzielt, indem sie die von Dirac erfundene „Methode der nochmaligen Quantelung" systematisch entwickelten. Neben gewissen sachlichen Schwierigkeiten, die viel tiefer liegen, trat dabei eine eigentümliche Schwierigkeit formaler Natur auf: der zum skalaren Potential kanonisch konjugierte Impuls verschwindet identisch, so daß die Aufstellung der Hamiltonschen Funktion und der Vertauschungsrelationen nicht ohne weiteres gelingt. Zur Beseitigung dieser Schwierigkeit sind bisher drei Methoden vorgeschlagen worden, die zwar ihren Zweck erfüllen, aber doch schwerlich als befriedigend betrachtet werden können.

1. Die erste Heisenberg-Paulische Methode ist ein rein analytischer Kunstgriff.[2]) Man fügt zur Lagrangefunktion gewisse Zusatzglieder hinzu, die mit einem kleinen Parameter ε multipliziert sind und bewirken, daß der obenerwähnte Impuls nicht mehr verschwindet. In den Schlußresultaten muß man dann zum Limes $\varepsilon = 0$ übergehen. Die ε-Glieder führen aber zu unphysikalischen Rechenkomplikationen[3]) und zerstören die charakteristische Invarianz der Lagrangefunktion gegenüber der Eichinvarianzgruppe.

2. Die zweite Heisenberg-Paulische Methode[4]) benutzt hingegen wesentlich diese Invarianz. Dem skalaren Potential

1) W. Heisenberg u. W. Pauli, Ztschr. f. Phys. **56**. S. 1. 1929; ebenda **59**. S. 168. 1930. Im folgenden mit H. P. I bzw. II zitiert.
2) H. P. I, S. 24—26, 30ff.
3) Vgl. L. Rosenfeld, Ztschr. f. Phys. **58**. S. 540. 1929.
4) H. P. II.

Annalen der Physik. 5. Folge. 5. 8

Rosenfeld's debt to Pauli

Bei der näheren Untersuchung dieser Verhältnisse an Hand des besonders lehrreichen Beispieles der Gravitationstheorie, wurde ich nun von Prof. Pauli auf das Prinzip einer neuen Methode freundlichst hingewiesen, die es in durchaus einfacher und natürlicher Weise gestattet, das Hamiltonsche Verfahren beim Vorhandsein von Identitäten auszubilden, ohne den Nachteilen der bisherigen Methoden ausgesetzt zu sein.

(As I was investigating these relations in the especially instructive example of gravitation theory, Professor Pauli helpfully indicated to me the principles of a simpler and more natural manner of applying the Hamiltionian procedure in the presence of identities. This procedure is not subject to the disadvantages of the earlier methods.)

Considers Lagrangians quadratic in field derivatives

$$\mathcal{L} = \frac{1}{2}\mathcal{A}^{\alpha\nu,\beta\mu}(Q)Q_{\alpha,\nu}Q_{\beta,\mu} + \cdots$$

$$\mathcal{L}_{em} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - eA_\mu\bar{\psi}\gamma^\mu\psi + i\hbar c\bar{\psi}\gamma^\mu\psi_{,\mu} - mc^2\bar{\psi}\psi$$

Considers coordinate transformations of the form

$$\delta x^\nu = a_r^\nu(x)\xi^r(x) + a_r^{\nu,\sigma}(x)\frac{\partial\xi^r}{\partial x^\sigma}$$

$$\delta x^\nu = 0$$

and gauge symmetry transformations of the form
(where the $\xi^r(x)$ are arbitrary)

$$\delta Q_\alpha = c_{\alpha r}(x,Q)\xi^r(x) + c_{\alpha r}^\sigma(x,Q)\frac{\partial\xi^r}{\partial x^\sigma}$$

$$\delta A_\mu = \delta_\mu^\sigma\xi_{,\sigma} \qquad \delta\psi = \frac{ie}{\hbar c}\psi\xi$$

Then suppose that Lagrangian transforms as a scalar density under
these symmetry transformations :

$$\delta\mathcal{L} + \mathcal{L}\frac{\partial\delta x^\mu}{\partial x^\mu} \equiv 0$$

$$\delta\mathcal{L}_{em} \equiv 0$$

The key observation: in the variation of the Lagrangian under symmetry transformations the coefficients of the highest time derivatives of the arbitrary functions ξ vanish identically.

First note that the canonical momentum is:

$$p^{\alpha} = \frac{\partial \mathcal{L}}{\partial \dot{Q}_{\alpha}} = \mathcal{A}^{\alpha\nu,\mu 0} Q_{\alpha,\nu}$$

$$\boxed{p^{\mu} = F^{0\mu} = E^{\mu}}$$

I -Then writing the relevant term in the variation we get primary constraints:

$$\delta \mathcal{L} = p^{\mu} c^{0}_{\mu r} \ddot{\xi}^{r} + \cdots \Rightarrow p^{\mu} c^{0}_{\mu r} \equiv 0$$

$$\boxed{\delta \mathcal{L}_{em} = -F^{00} \ddot{\xi} + \cdots \Rightarrow p^{0} \equiv 0}$$

II - Note also that the primary constraints give us null vectors of the Legendre matrix :

$$\mathcal{A}^{\alpha 0,\mu 0} c^{0}_{\mu r} \equiv 0$$

$$\boxed{\mathcal{A}^{\alpha 0,\mu 0} \delta^{0}_{\mu} \equiv 0 \equiv \mathcal{A}^{\alpha 0,00}}$$

Lagrangian doesn't depend on $\dot{A}_0$

III - Since $p^{\alpha} = \mathcal{A}^{\alpha 0,\mu 0} \dot{Q}_{\mu} + \cdots$ velocities are not uniquely fixed in terms of the momenta:

$$\dot{Q}_{\mu} = \frac{\partial \mathcal{H}_0}{\partial p^{\mu}} + \lambda^{r} c^{0}_{\mu r} = \frac{\partial \left(\mathcal{H}_0 + \lambda^{r} p^{\nu} c^{0}_{\nu r}\right)}{\partial p^{\mu}}$$

$$\boxed{\dot{A}_{\mu} = \frac{\partial \left(p^{b} p_{b} - p^{b} A_{0} + \lambda p^{0}\right)}{\partial p^{\mu}}}$$

λ^{r} are arbitrary functions

The canonical generator of gauge transformations

First Rosenfeld proved that the following integral generates the correct gauge variations

$$\overline{\mathcal{M}} = \int d^3x \, p^\alpha \delta Q_\alpha$$

$$\overline{\mathcal{M}_{em}} = \int d^3x \left(p^\mu \xi_{,\mu} - e\psi^* \psi \xi\right)$$

Rosenfeld then proved that this generator could always be written as the sum of time derivatives of the primary constraints multiplying time derivatives of the arbitrary functions ξ^r, and must therefore vanish.

$$\overline{\mathcal{M}} = \int d^3x \left(\frac{d\xi^r}{dt} p^\mu c^0_{\mu r} - \xi^r \frac{d}{dt}\left(p^\mu c^0_{\mu r}\right)\right)$$

$$\overline{\mathcal{M}_{em}} = \int d^3x \left(p^0 \dot{\xi} - \left(E^a_{,a} - e\psi^* \psi\right) \xi\right)$$

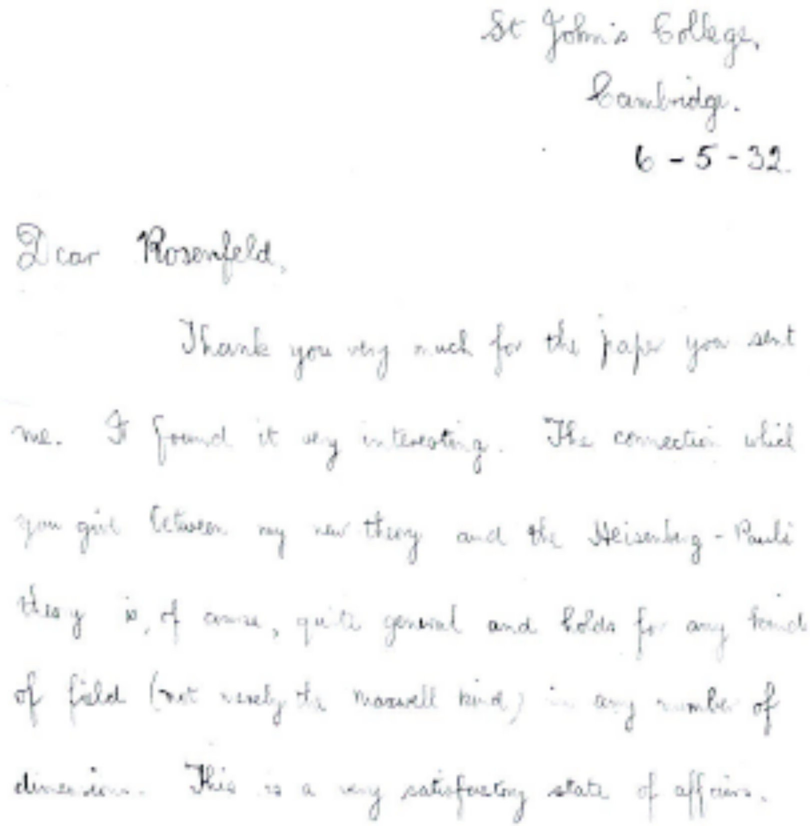
In summary - Rosenfeld has shown how gauge symmetry transformations can be implemented within a canonical Hamiltonian framework. This formalism is called “constrained Hamiltonian dynamics, and is commonly known as the Dirac-Bergmann procedure. It provides the mathematical proof of the legitimacy of both the Lorentz gauge and the Coulomb gauge.

6. Rosenfeld and Dirac

Dirac to Rosenfeld, 4/26/31: Many thanks for sending a copy of your paper on radiation theory, which I have read with great interest. (Niels Bohr Archive)

Rosenfeld to Dirac, 4/30/32: I enclose a note about your new theory, which is clearly not at all meant "um zu kritisieren" but "nur um zu lernen". (Churchill College Archive)

Rosenfeld publishes demonstration of equivalence of Heisenberg-Pauli and Dirac many-body theory in 1932 - submitted May 2.



St John's College,
Cambridge.
6 - 5 - 32.

Dear Rosenfeld,

Thank you very much for the paper you sent me. I found it very interesting. The connection which you give between my new theory and the Heisenberg - Pauli theory is, of course, quite general and holds for any kind of field (not simply the Maxwell kind) in any number of dimensions. This is a very satisfactory state of affairs.

Thank you very much for the paper you sent me. I found it very interesting. The connection which you give between my new theory and the Heisenberg - Pauli theory is, of course, quite general and holds for any kind of field (not simply the Maxwell kind) in any number of dimensions. This is a very satisfactory state of affairs. (Niels Bohr Archive)

Dirac published an "improved" demonstration with Fock and Podolsky later in 1932

Rosenfeld to Dirac, (5/10/32) … As to the doubtful sentence of Heisenberg-Pauli, which you are right in not understanding, I would suggest to you to examine the general invariance proof which I give in my paper of the "Annalen der Physik", 5, 113, 1930. (I sent you reprints of both). (Niels Bohr Archive)

I have been studying your papers, but have had some trouble in understanding the significance of your λ's. What exactly is meant by the statement that they are arbitrary? On page 143 of your Annalen paper, the first of equations (III), when worked out, gives

Dirac to Rosenfeld, (5/16/32) … I have been studying your papers, but have had some trouble in understanding the significance of your λ's. What exactly is meant by the statement that they are arbitrary? (Niels Bohr Archive)

Rosenfeld to Dirac, (5/21/32) … As to the λ's, they enter as arbitrary or undetermined coefficients (depending on coordinates) in the general expression of the $\dot{Q}$ *in terms of the Q's and P's. In equation (111) the hamiltonian should be the same as that of Heisenberg-Pauli (as stated there), so that the substitution of the P's in terms of the* $\dot{Q}$ *in them will lead to identities, and this implies no restriction for λ (Churchill College Archive)*

6. The impact of Rosenfeld's work on constrained Hamiltonian dynamics.

- Pauli to O. Klein (1/25/55)

Gerne möchte ich Dich in dieser Verbindung auf die lange Arbeit von Rosenfeld, Annalen der Physik (4), 5, 113, 1930 aufmerksam machen. Er hat sie seinerzeit bei mir in Zurich gemacht und hiess hier dementsprechend ``der Mann, der das Vierbein quantelt'' (klingt wie der Titel eines Grimmschen Märchens, nicht?). - Siehe dazu Teil II seiner Arbeit, wo das ``Vierbein'' daran kommt. Auf die Identitäten zwischen den "p" und "q"' - d.h. kanonisch konjugierten Feldern - die eben aus der Existenz der Gruppe der Allgemeinen Relativitätstheorie (Koordinaten-Transformationen mit 4 willkürlichen Funktionen) entspringen, wurde damals besonderer Wert gelegt. Ich erinnere mich noch, dass Rosenfelds Arbeit nicht in jeder Hinsicht befriedigend war, da er gewisse zusätzliche Bedingungen einführen musste, die niemand richtig verstehen konnte.

(I would like to bring to your attention the work by Rosenfeld in 1930. He was known here at the time as the `man who quantised the Vierbein' (sounds like the title of a Grimm's fairy tale doesn't it?) See part II of his work where the Vierbein appears. Much importance was given at that time to the identities among the p's and q's (that is the canonically conjugate fields) that arise from the existance of the group of general coordinate transformations. I still remember that I was not happy with every aspect of his work since he had to introduce certain additional assumptions that no one was satisfied with.

A perhaps pertinent remark of Dirac

…Well, I think I might answer you in much the same way that I wrote that I felt it had probably been done before, but it was less trouble to me to present it as something new than to search for a reference. A good deal of my work was like that. It happened rather often that there was something which I thought had been done before, but it seemed a great nuisance to look through all the references to try to find it, and if it doesn't take much trouble to publish it, one can publish it again without claiming either that it is new or that it has been done before. (AHQM, 5/10/63, p 15)