Fully relative general relativity

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See arXiv 1508.01277 for some of this material and references

Overview

1. INTRODUCTION

Introduction

Focus of this talk:

- The use of the four-dimensional diffeomorphism group to construct evolutions with respect to temporal and spatial landmarks

Questions to be addressed:

- Why were early successes in the Hamiltonian realization of general covariance symmetry abandoned?
- Why and how has the Dirac non-covariant approach dominated canonical approaches to quantum gravity?
- How can the fully relational approach be applied to loop quantum gravity?

2. BRIEF HISTORY OF EARLY HAMILTONIAN GENERAL COVARIANCE

Rosenfeld's 1930 tetrad gravitational Lagrangian

"Zur Quantelung der Wellenfelder", *Annalen der Physik* **397**, 113 (1930)

$$\mathcal{L} = \frac{1}{2\kappa}(-g)^{\frac{1}{2}} E^{\mu}_{I} E^{\nu}_{J} \left({\omega_{\mu}}^{I}{}_{L} {\omega_{\nu}}^{LJ} - {\omega_{\nu}}^{I}{}_{L} {\omega_{\mu}}^{LJ} \right)$$
$$+\Re\left\{ (-g)^{1/2} \left[\frac{1}{2} i\bar{\psi}\gamma^{\mu} \left(\overrightarrow{\partial}_{\mu} + \Omega_{\mu} \right) \psi - m\bar{\psi}\psi \right] \right\} + \mathcal{L}_{em}$$

Rosenfeld's 1930 tetrad Hamiltonian density

$$\mathcal{H} = \mathcal{H}_0\left[g_{ab}, p^{ab}, A_a, p^a, \psi, \psi^\dagger\right] + \lambda_I \mathcal{F}^I + \lambda_{IJ}\mathcal{F}^{[IJ]} + \lambda \mathcal{F}$$

where $\mathcal{F}^I$, $\mathcal{F}^{[IJ]}$ and $\mathcal{F}$ are primary constraints and λ_I, λ_{IJ} and λ are arbitrary spacetime functions.

Felix Klein, Emmy Noether and Wolfgang Pauli

Felix Klein was the first to systematically deduce identities that followed from the fundamental identity expressing the transformation properties of a Lagrangian under local symmetries

Nachrichten von der Königl. Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse, **2**, 171 - 189, 1918

Emmy Noether's second theorem was essentially the derivation of the contracted Bianchi identities

Nachrichten von der Königl. Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse, **2**, 235 - 257 (1918).

Felix Klein, Emmy Noether and Wolfgang Pauli

Felix Klein launched the Encyclopedia of Mathematical Science, invited Wolfgang Pauli to write his 1921 relativity review contribution, and critically reviewed Pauli's drafts.

It is likely that Pauli brought Klein's algorithm to Rosenfeld's attention. It became the basis of Rosenfeld's derivation of phase space generators of infinitesimal symmetry transformations.

Rosenfeld's infinitesimal phase space symmetry generator

Rosenfeld proved that the vanishing Noether charge generated the correct variations of all of the phase space variables under all of the local symmetries. Most importantly for us is that the active variations under the infinitesimal coordinate transformations $x'^{\mu} = x^{\mu} + \xi^{\mu}(x)$ are correct.

His conserved and vanishing generating density is

$$-\mathcal{F}^I e_{0I}\dot{\xi}^0 - \mathcal{F}^I e_{aI}\dot{\xi}^a - \mathcal{F} A_0 \dot{\xi}^0 - p^{aI} e_{\nu I}\xi^{\nu}_{,a} - p^a A_{\nu}\xi^{\nu}_{,a} - \mathcal{H} A_0 \xi^0 - \mathcal{G}_a \xi^a$$

$$-\mathcal{F}\dot{\xi} + p^a \xi_{,a} + i\frac{e}{\hbar c} p_{\psi}\psi\xi - i\frac{e}{\hbar c} p_{\psi^\dagger}\psi^\dagger\xi + \mathcal{F}_{[IJ]}\xi^{IJ} = 0$$

Peter Bergmann and Paul Dirac

We leap two decades forward to the contributions to constrained Hamiltonian dynamics of Peter Bergmann and Paul Dirac - beginning in 1949.

Dirac never concerned himself with the phase space realization of the full general covariance group. See his Vancouver lectures, *Canadian Journal of Mathematics*, **2**, 129 - 148 (1950) and **3**, 1 - 23 (1951)

Bergmann (1949), with Jim Anderson (1951) and numerous collaborators did concern themselves with this symmetry. In particular a joint publication with Ralph Schiller explicitly employed the vanishing Noether charge.

Bergmann and Schiller, *Physical Review* **89**, 4 - 16, (1953)

Non-realizability of the diffeomorphism Lie algebra

But there is an obstacle to the realization of finite diffeomorphisms, explicitly recognized by Bergmann. One can see this in the Lie algebra

$$\xi_3^\mu = \xi_{1,\nu}^\mu \xi_2^\nu - \xi_{2,\nu}^\mu \xi_1^\nu = \xi_{1,0}^\mu \xi_2^0 - \xi_{2,0}^\mu \xi_1^0 + \dots$$

Repeated commutators lead to higher and higher order time derivatives.

Dirac's resolution

Dirac's solution was probably inspired by his student Paul Weiss: write the infinitesimal variations as a sum of perpendicular and tangent increments,

$$\xi^\mu = n^\mu \epsilon^0 + \delta^\mu_a \epsilon^a$$

This results in the familiar metric dependent Dirac algebra.

The Bergmann - Komar group

Bergmann and Komar interpreted this algebra as representing a compulsory metric-dependent transformation group

International Journal of Theoretical Physics, **5**, 15 - 28, (1972)

3. LEGENDRE PROJECTABILITY AND THE INDUCED DIFFEOMORPHISM TRANSFORMATION GROUP

The Legendre projectability requirement

We have an mathematical justification for the Dirac decomposition. It is required in order that configuration-velocity variations be projectable under the Legendre transformation to phase space.

The generator of infinitesimal transformations

Confining our attention to general coordinate transformations the Noether generator density is

$$\mathcal{G}_\epsilon(t) = P_\mu \dot{\epsilon}^\mu + (\mathcal{H}_\mu + \int d^3x' \int d^3x'' N^{\rho'} C^{\nu''}_{\mu\rho'} P_{\nu''})\epsilon^\mu .$$

where

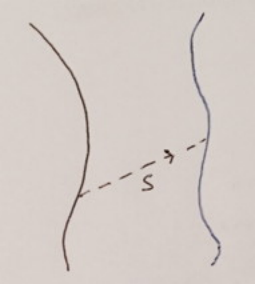
$$\{\mathcal{H}_\mu(x), \mathcal{H}_\rho(x')\} = C^{\nu''}_{\mu\rho'} [g_{ab}] \mathcal{H}_{\nu''}$$

Time evolution versus diffeomorphisms

The evolution in time is generated by

$$H = \int d^3x \left(N\mathcal{H}_0 + N^a \mathcal{H}_a + \lambda_\mu P^\mu \right).$$

The finite diffeomorphism generator $\exp\left(s \int d^3x\, \mathcal{G}_\epsilon(t)\right)$ transforms solutions into new solutions.



Enlargement of phase space

Note that the lapse function N and shift N^a must be retained as canonical variables.

Note also that contrary to popular belief, the Hamiltonian formulation does not fix a time foliation. New foliations result in new multipliers λ^μ and new Hamiltonians as a consequence of the time dependence of the Hamiltonian.

4. CLASSICAL INTRINSIC DYNAMICS AND NATURAL WHEELER DEWITT EQUATIONS

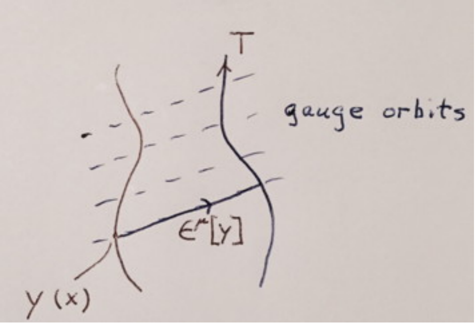
The implementation of Carlo's partial variable program

Now that we have the full diffeomorphism group at our disposal, we can employ it to establish correlations between partial variables. One possible implementation, in principle, is to locate temporal and spatial landmarks by referring to curvature even in the vacuum case. There are of course many more possibilities when matter is present. We will employ these landmarks as "intrinsic" coordinates. Such coordinates must be formed from spacetime scalars. Thus we choose $X^\mu[g_{ab}, p^{ab}]$.

In the vacuum case we propose the use of the four Weyl curvature scalars, as originally suggested by Komar in the 1950's. They are quadratic and cubic in the Weyl tensor. Bergmann and Komar showed in 1960 that they are expressible solely in terms of the three metric and its conjugate momenta.

Intrinsic coordinate gauge conditions

We choose intrinsic coordinates through the gauge conditions $x^\mu = X^\mu[g_{ab}, p^{ab}]$. Given any solution trajectory in phase space we can then determine the phase space dependent finite descriptors $\epsilon^\mu[g_{ab}, p^{ab}] := \epsilon^\mu[y]$ that will gauge transform these solutions to those that satisfy the gauge conditions.



The explicit construction of evolving constants of the motion

This construction yields Taylor expansions in the coordinates x^μ - now themselves diffeomorphism invariants. The coefficients in the Taylor expansions are functionals of g_{ab} and p^{ab} that are explicitly diffeomorphism invariants. This applies also to the invariant lapse and shift.

$$\mathcal{I}_\phi = \sum_{n_\mu=0}^{\infty} \frac{1}{n_0!\, n_1!\, n_2!\, n_3!} (x^0)^{n_0} (x^1)^{n_1} (x^2)^{n_2} (x^3)^{n_3}\, \mathcal{C}_{n_0,n_1,n_2,n_3}[g_{ab}, p^{ab}]$$

Kuchar-inspired canonical transformations

Canonical transformations can be carried out to new canonical variables including X^μ and canonical conjugates π_μ - but without imposing gauge conditions. The theory in terms of these new variables is still fully diffeomorphism covariant - with corresponding Hamiltonian constraints. Each choice yields a new form for the constraints and a new Wheeler-DeWitt equation with a corresponding "natural" choice of temporal and spatial partial variables.

This "natural" choice is the one that results through the solutions of the Wheeler-DeWitt equation.

Fully relative general relativity

Claim: The range of intrinsic coordinates is coincident with the set of coordinates obtained under arbitrary coordinates transformations

Corollary: For every choice of coordinate chart there is a corresponding choice of intrinsic coordinates.

5. IMPLICATIONS FOR LOOP QUANTUM GRAVITY?

Intrinsic coordinates for loop quantum gravity?

My suspicion is that a “natural” choice of intrinsic coordinates appears in the construction of the spin network kinematical framework, scalar constraint Hamiltonian regularization schemes, and in the Y_γ map in the covariant approach.

References I