QUANTUM GENERAL INVARIANCE

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A quantum physical projector is proposed for generally covariant theories which are derivable from a Lagrangian. The projector is the quantum analogue of the integral over the generators of finite one-parameter subgroups of the gauge symmetry transformations which are connected to the identity. Gauge variables are retained in this formalism, and phase space is enlarged by interpreting time as a variable label of the arbitray configuration space gauge functions.

1 Introduction

In a recent series of papers I and my collaborators Josep Pons and Larry Shepley have shown how symmetries induced by diffeomorphism gauge symmetries in a wide class of classical generally covariant Lagrangian models are retained in the Hamiltonian formulation of these models. [1,2,3,4] We look for configuration-velocity symmetry variations which are projectable onto the classical phase space. In most cases the resulting condition is that symmetry variations may not depend on time derivatives of the gauge functions, which for the Ashtekar formulation of general relativity are the lapse, shift, and temporal component of the Ashtekar connection. As a consequence projectable spacetime diffeomorphism symmetries acquire a compulsory dependence on the gauge variables, and when additional gauge symmetries are present as in the Ashtekar formalism, projectable spacetime diffeomorphism-related symmetries necessarily include these additional symmetries. Quite generally, the symmetry group becomes a transformation group on the dynamical variables, including the gauge variables, and in this sense is similar to internal gauge groups of the Yang-Mills type.

One important outcome of this analysis is that time evolution in the phase space formulation of generally covariant theories is *not* a gauge symmetry; an element of the gauge symmetry group will generally effect a rigid translation in time only on one member in an equivalence class of solution trajectories. The implications for the construction of both classical and quantum symmetry invariants are profound; invariants need not in general be time independent. An additional significant dividend, related to the retention of space-time diffeomorphism-related symmetries, is that the constrained physical phase space in the theory includes the gauge variables. Thus, although a time foliation of the spacetime manifold is assumed in the phase space formulation, the symmetry group effectively generates all foliations. This result is achievable because the gauge variables are retained, and under arbitrary spacetime diffeomorphism related transformations, they and the conventional non-gauge variables intermix. This is truly a desireable property of an eventual quantum theory of gravity for it should render possible the construction of true spacetime invariants such as four-dimensional spacetime volumes and time-like 2- and 3-areas. At this time only space-like 2- and 3-areas have been shown to be quantized in loop approaches to quantum gravity based on the Ashtekar connection. [5]

2 The physical projector

I propose to retain the gauge variables as quantum operators. Since they are indeed arbitrary functions of time I suggest that for them time be interpreted as a variable label; considering, for example, the classical lapse function $N(t,\vec{x})$, with associated canonical momentum $\pi(t,\vec{x})$, which classically is constrained to vanish, we assume Poisson brackets $\{N(t,\vec{x}),\pi(t',\vec{x}')\} = \delta(t,t')\delta^3(\vec{x},\vec{x}')$. I have shown elsewhere that both the generators of time evolution and symmetry variations may be written in terms of these new phase space variables. One need only check that the generators produce the correct variations assuming this new Poisson bracket algebra. [6]

Quantum general invariants will then be constructed, in a manner suggested by Rovelli [7], by averaging over the finite one-parameter symmetry subgroups built up from the infinitesmal generators. At this time the program has been implemented only partially on the relativistic free particle, and I will sketch the results here. [6] Letting θ represent the parameter fixing an event on the particle world line, the theory is covariant under infinitesmal reparametrizations which depend on the induced metric, which is the lapse $N(\theta)$: $\theta' = \theta - \xi(\theta)dsN(\theta)^{-1}$, where $\xi(\theta)$ is an arbitrary finite function, and ds is an infinitesmal subgroup parameter. This N dependent general parameter transformation induces a variation in both the scalar lapse and the scalar descriptor ξ. Integration of these variations results in the following formal expression for the family of lapses $N_s(\theta)$ generated by the one parameter symmetry subgroup parametrized by s:

$$N + \sum_{n=0}^{\infty} \frac{1}{(n+1)!}\frac{d}{d\theta}\left(\xi N^{-1}\frac{d}{d\theta}\left(\ldots\frac{d}{d\theta}\left(\xi N^{-1}\frac{d\xi}{d\theta}\right)\ldots\right)\right)s^{n+1}, \tag{1}$$

where ξN^{-1} appears $n-1$ times. N and ξ are the original lapse and descriptor, respectively.

It is straightforward to find the classical phase space generator of $N_s(\theta)$, in terms of the canonical variables $N(\theta)$ and $\pi(\theta)$. The quantum physical projector is the functional integral of the quantum version of this generator over the descriptor function $\xi(\theta)$.

References

1. J. M. Pons, D. C. Salisbury, and L. C. Shepley, *Phys. Rev.* D **55**, 658 (1997) gr-qc/9612037
2. J. M. Pons, D. C. Salisbury, and L. C. Shepley, *J. Math. Phys.* **41**, 5557 (2000) gr-qc/9912086
3. J. M. Pons, D. C. Salisbury, and L. C. Shepley, *Gen. Rel. Grav.* **32**, 1727 (2000) gr-qc/9912087
4. J. M. Pons, D. C. Salisbury, and L. C. Shepley, *Phys. Rev.* D **62**, 064026 (2000) gr-qc/9912085
5. C. Rovelli and L. Smolin, *Nucl. Phys.* B **442**, 593 (1995) gr-qc/9411005
6. D. C. Salisbury, "Quantum general invariance and loop gravity", submitted for publication
7. C. Rovelli, *Phys. Rev.* D **59**, 104015 (1999) gr-qc/9806121