Einstein, Gravity, and the Search for a Unified Field Theory

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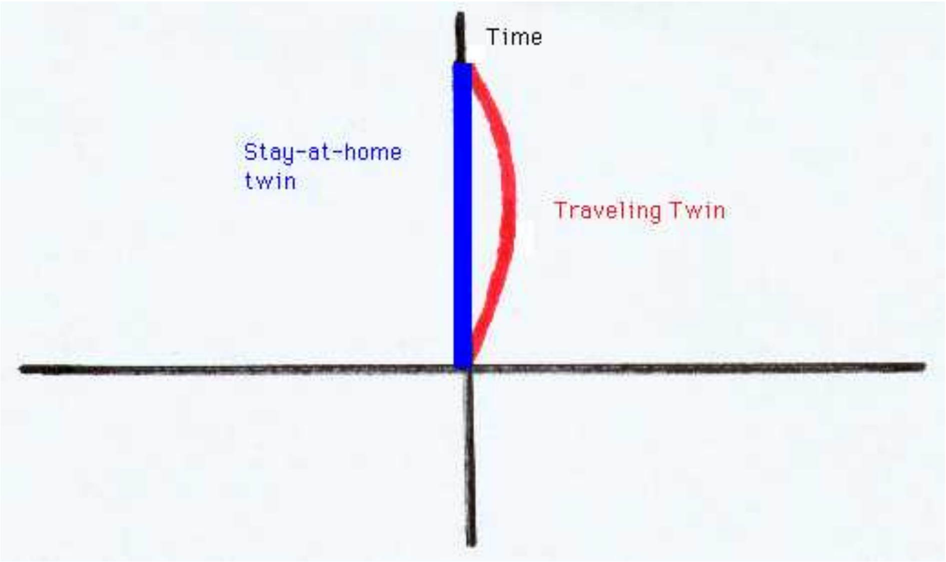
Plan of Talk

1. Flat spacetime metric and the twin paradox
2. Equivalence principle and Einstein's geometric insight
3. Local flatness
4. Tidal forces and "derivation" of vacuum Einstein equations
5. Black holes
6. Cosmology and Einstein equations with material sources
7. Einstein and Peter Bergmann
8. Unification of gravity and electromagnetism

1 - Flat spacetime metric and the twin paradox

- Flat spacetime metric yields elapse of proper time
- There is no twin "paradox"



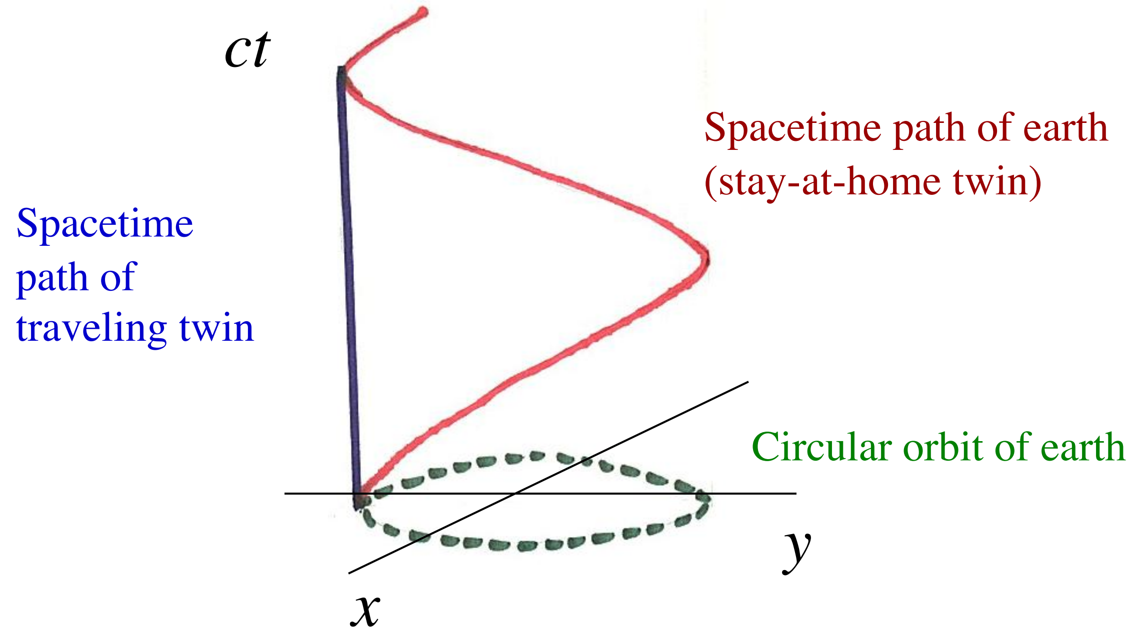
$$c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

Red twin ages less than blue twin

Einstein's interpretation: "spacetime distance" along red path is less than along blue path

2 - Equivalence principle and Einstein's geometric insight

- All objects released from rest near earth's surface move along the same spacetime trajectory
- Einstein interprets as "straightest" path in curved spacetime



Earth in orbit around sun: longest ("straightest") spacetime path is helical

3 - Local flatness

- Can introduce almost Cartesian coordinates on the two-dimensional surface of a sphere

$$\text{Try } X = R\phi \text{ and } Y = R\left(\frac{\pi}{2} - \theta\right)$$

$$ds^2 = R^2 d\theta^2 + R^2 \sin^2(\theta) d\phi^2 = \cos^2\left(\frac{Y}{R}\right) dX^2 + dY^2 \approx \left(1 - \frac{Y^2}{R^2}\right) dX^2 + dY^2$$

- Deviation from flatness is of second order in the coordinates

Parallel transport around a closed curve

- Parallel transport maintains constant angle with respect to geodesic
- Geodesic that starts at $(0,\Delta Y)$ and ends at $(\Delta X,\Delta Y)$ must have initial direction

$$\left.\frac{dY}{dX}\right|_{X=\Delta X} = \frac{\Delta X \Delta Y}{2R^2} = \frac{\Delta\theta\Delta\phi}{2} \equiv \alpha$$

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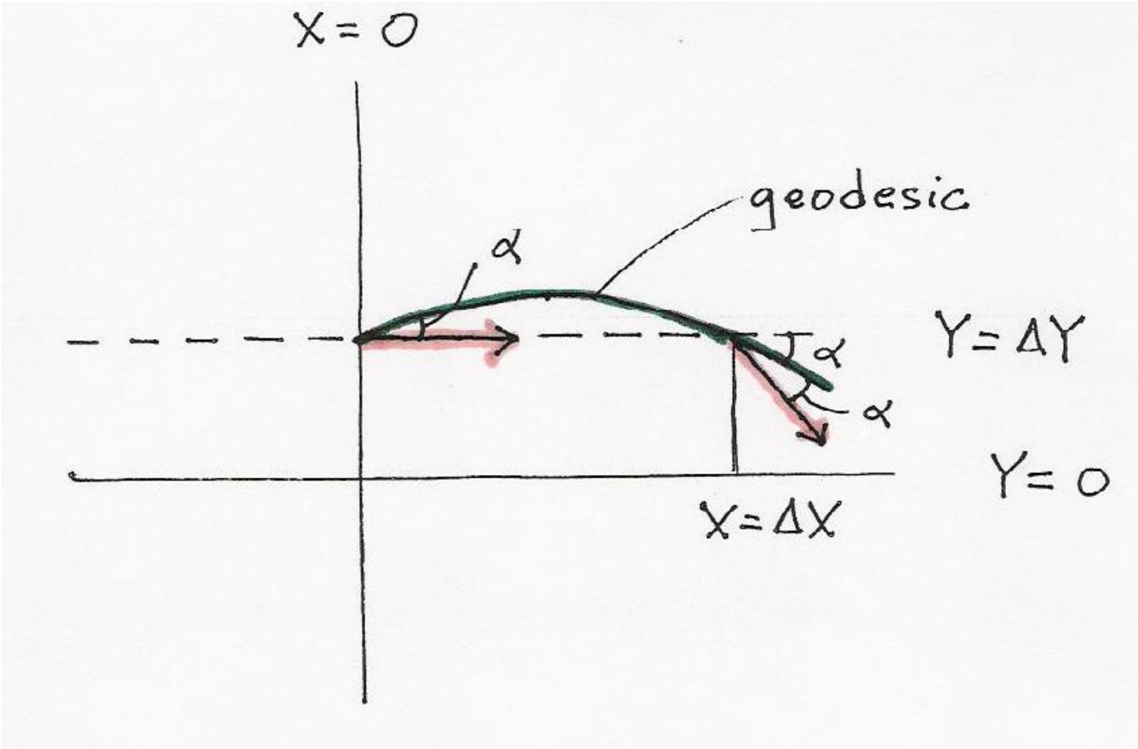
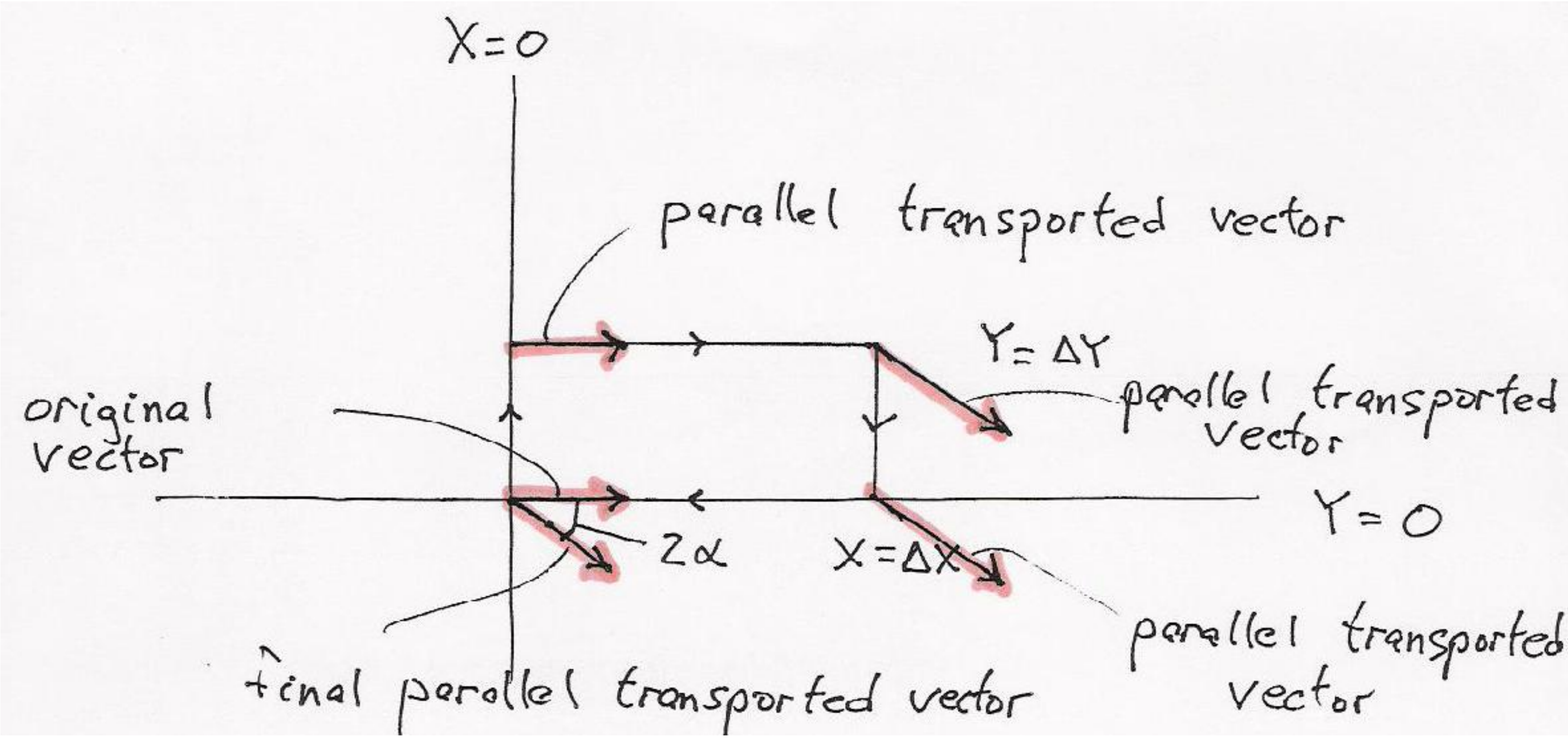
- Total angle of rotation around closed path is 2α
- Curvature is defined to be net deviation angle divided by the area

$$\text{Curvature} = \frac{2\alpha}{R^2 \Delta\theta \Delta\phi} = \frac{1}{R^2}$$




X=0
parallel transported vector
Y= ΔY
original vector
parallel transported vector
Y=0
2α
X=ΔX
parallel transported vector
final parallel transported vector

4 - Tidal Forces and "derivation" of Einstein equations

Newtonian gravitational equations of motion are the extremal paths of the spacetime metric

$$c^2 d\tau^2 = \left(1 + 2\Phi(\vec{r})\right) c^2 dt^2 - \left(dx^2 + dy^2 + dz^2\right)$$

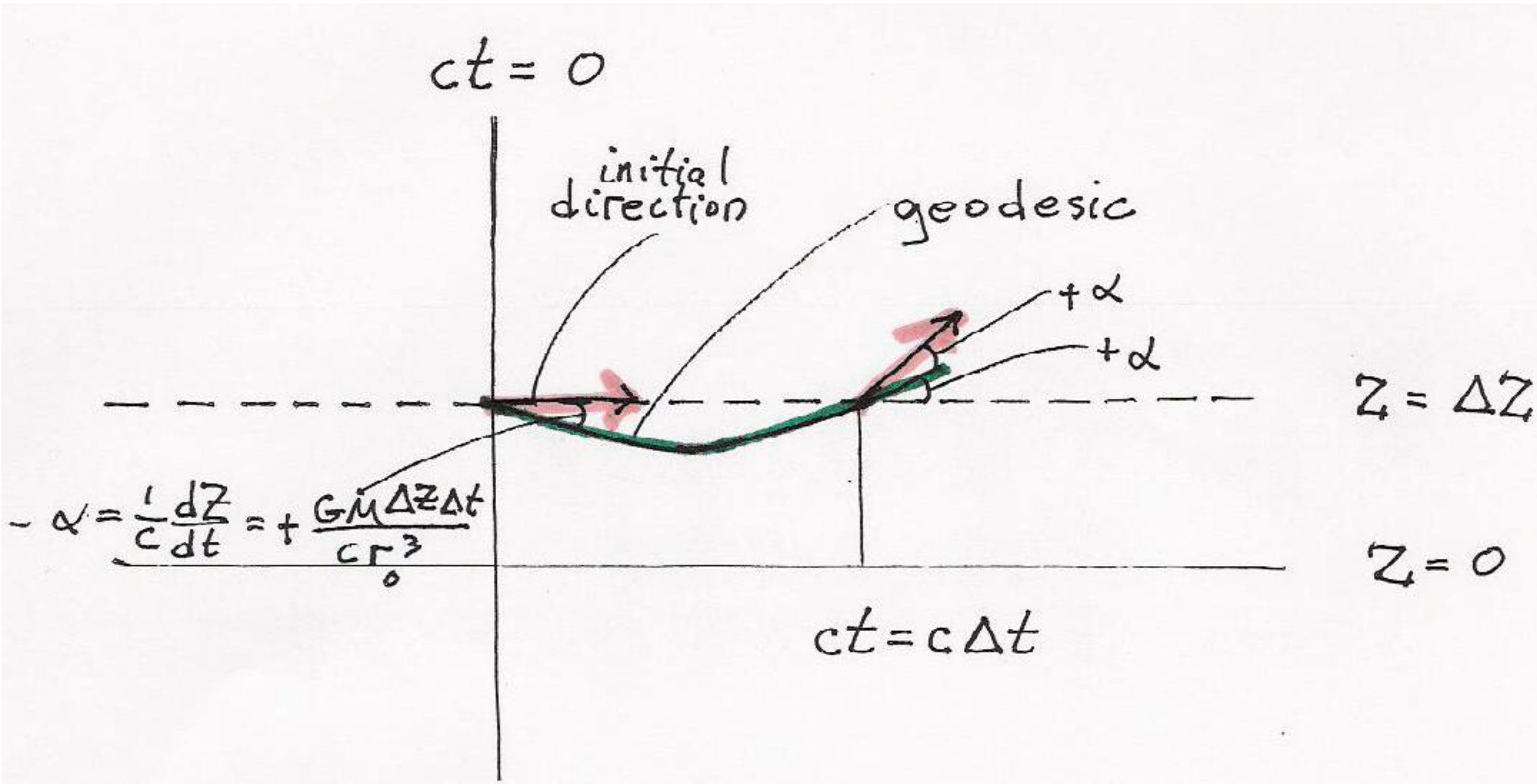
$\Phi(\vec{r})$ is the Newtonian gravitational potential energy

The gravitational potential energy in a free-falling frame of reference near the surface of the earth is

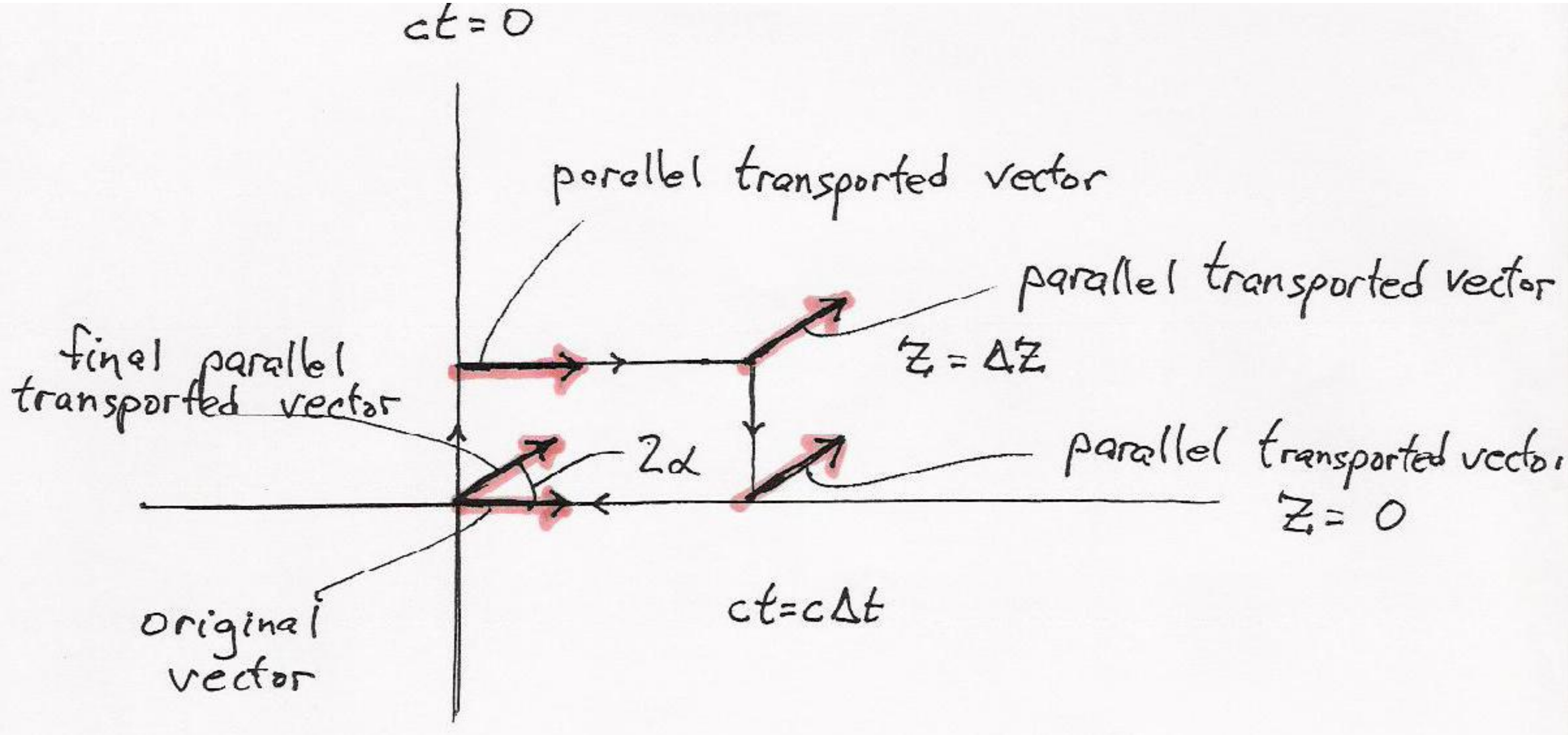
$$\Phi = \frac{GM\left(X^2 + Y^2 - 2Z^2\right)}{r_0}$$

The spacetime metric in these coordinates is locally flat (local inertial frame of reference)

- Consider parallel transport in the *t*-*Z* plane



- Parallel transport around a closed path in *t-Z* plane



Net rotation of a vector in the *t-Z* for transport around a path in the *t-Z* plane is

$$2\alpha = \frac{2GM\Delta Z\Delta t}{cr_0^3}$$

Curvature is $$\frac{1}{R^2} = \frac{2\alpha}{\Delta Zc\Delta t} = \frac{2GM}{c^2 r_0^3}$$

- Similar calculation gives curvatures is *t*-*X* and *t*-*Y* planes:

$$-\frac{GM}{c^2 r_0^3}$$

- Notice that the sum of the three curvatures is zero

- Since there is no material source within this freely falling reference frame, this suggests that the sum of the curvatures should generally be set equal to zero for vacuum spacetimes

- This implies there must be curvature also in space. A calculation yields

$$c^2 d\tau^2 = \left(1 + 2\Phi(\vec{r})\right) c^2 dt^2 - \left(1 - 2\Phi(\vec{r})\right) dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

5 - Blackholes

- Blackhole metric

$$c^2 d\tau^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 - r^2\left(d\theta^2 \sin^{2+} \theta d\phi^2\right)$$

  - Time slows near the blackhole horizon

6 - Cosmology and Einstein equations with material sources

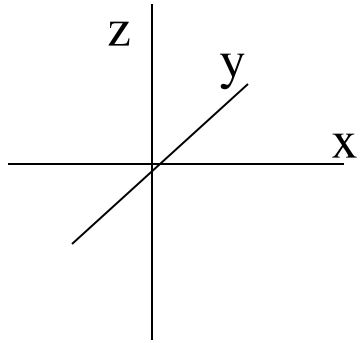
- Friedmann-Robertson-Walker metric

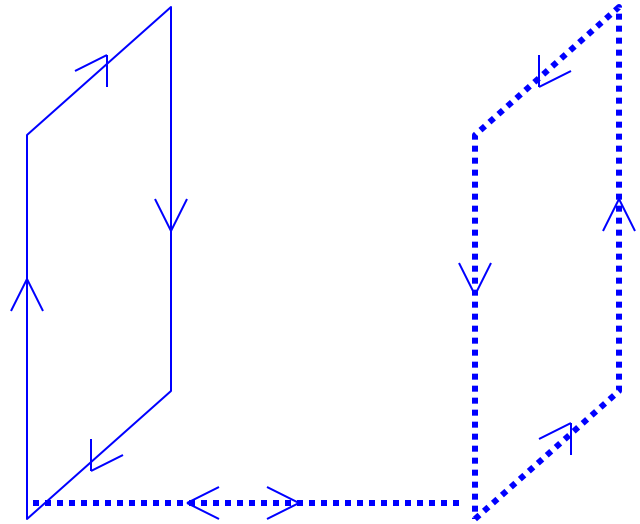
$$c^2 d\tau^2 = c^2 dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 \sin^{2+}\theta\, d\phi^2\right)\right)$$

- Friedmann-Robertson-Walker metric in local inertial coordinates (where the inertial time is

$$T = t - t_0 + \frac{1}{2c^2} a(t)\dot{a}(t) r^2$$

$$c^2 d\tau^2 = c^2 dT^2 - dX^2 - dY^2 - dZ^2 - \frac{\ddot{a}_0}{a_0}\left(X^2 + Y^2 + Z^2\right) dT^2 - \left(\frac{\dot{a}_0^2}{a_0^2 c^2} + \frac{k}{a_0^2}\right)\left(XdX + YdY + ZdZ\right)^2$$

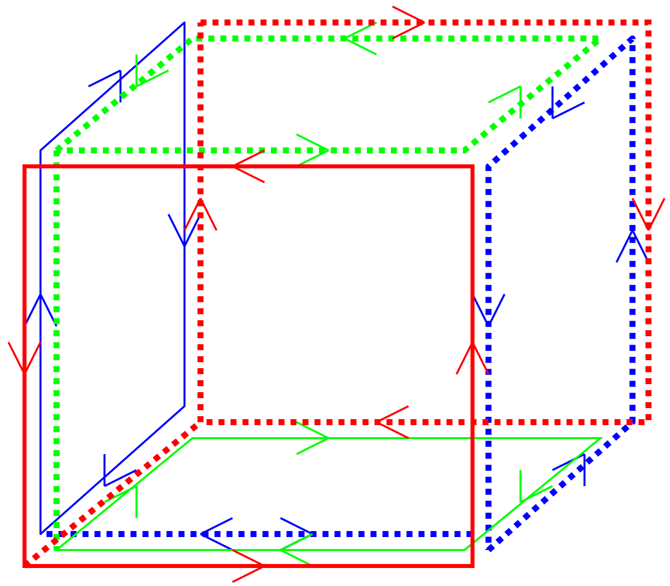




$R^{\alpha}{}_{\beta yz}(x+\Delta x, y, z)$ rotation matrix for parallel transport around loop in *y*-*z* plane at $x+\Delta x$

$\dfrac{\partial R^{\alpha}{}_{\beta yz}(x, y, z)}{\partial x}$ net rotation matrix for parallel transport about edges of both faces of cube

Add transport around edges of remaining four faces:



Each edge is traversed once in each direction! The net path goes nowhere!

Bianchi identity: $$\frac{\partial R^{\alpha}{}_{\beta yz}(x,y,z)}{\partial x} + \frac{\partial R^{\alpha}{}_{\beta zx}(x,y,z)}{\partial y} + \frac{\partial R^{\alpha}{}_{\beta xy}(x,y,z)}{\partial z} \equiv 0$$

- Contracted Bianchi identity gives left hand side of Einstein equations consistent with conservation of energy-momentum

$$T^{\alpha\beta} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

$$\sum_{\beta=0}^{3} \frac{\partial T^{\alpha\beta}}{\partial x^{\beta}} = 0$$

$$\sum_{\alpha=0}^{3} R^{\alpha}_{\beta[\alpha\gamma,\delta]} \equiv 0$$

- Einstein cosmological equations

$$\left(\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a} \right) = \frac{8\pi G}{3c^2}\rho + \frac{\Lambda c^2}{3}$$

$$-2\frac{\ddot{a}}{ac^2} - \left(\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a} \right) = \frac{8\pi G}{c^4} p - \Lambda$$

Einstein and Peter Bergmann

Fanciful Interlude

Bergmann’s early life

- Born Berlin-Charlottenburg 1915
- Mother Dr. Emmy Bergmann moved with children to Freiburg 1921 - she and sister emigrated to Israel 1935
- Father Dr. Max Bergmann 1921 - 1933 head of Institut für Lederforschung, Dresden (now Max Bergmann Zentrum für Biomaterialen)
- Prague, Charles University degree 1936

Einstein-Bergmann collaboration and Syracuse University

- 1936 - 1941: unified field theory
  - Unification of gravity and electromagnetism
  - Reality of fifth dimension
  - Relation of coordinate transformations in fifth dimension to symmetries of electromagnetism
- Syracuse University 1947 - 1982
- Early Publications
  - *Introduction to the Theory of Relativity* 1942
  - *Basic Theories of Physics: Mechanics and Electromagnetism* 1949
  - *Basic Theories of Physics: Heat and Quanta* 1951

- Kaluza metric

$$c^2 d\tau^2 = \sum_{\mu,\nu=0}^{3} \left(g_{\mu\nu}(x^\alpha) + A_\mu A_\nu \right) dx^\mu dx^\nu + \sum_{\mu=0}^{3} A_\mu dx^\mu dx^5 + dx^5 dx^5$$