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Peter Bergmann and the Invention of Constrained Hamiltonian Dynamics

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11.1 Introduction

It has always been the practice of those of us associated with the Syracuse "school" to identify the algorithm for constructing a canonical phase space description of singular Lagrangian systems as the Dirac–Bergmann procedure. I learned the procedure as a student of Peter Bergmann, and I should point out that he never employed that terminology. Yet it was clear from the published record at the time (the 1970s) that his contribution was essential. Constrained Hamiltonian dynamics constitutes the route to canonical quantization of all local gauge theories, including not only conventional general relativity, but also grand unified theories of elementary particle interactions, superstrings, and branes. Given its importance and my suspicion that Bergmann has never received adequate recognition from the wider community for his role in the development of the technique, I have long intended to explore this history in depth. This paper is merely a tentative first step, in which I will focus principally on the work of Peter Bergmann and his collaborators in the late 1940s and early 1950s, indicating where appropriate the relation of this work to later developments. I begin with a brief survey of the prehistory of work on singular Lagrangians, followed by some comments on the life of Peter Bergmann. These are included in part to commemorate Peter in this first meeting on the History of General Relativity since his death in October 2002. Then I will address what I perceive to be the principal innovations of his early Syracuse career. Josh Goldberg has already covered some of this ground in his 2005 report (Goldberg 2005), but I hope to contribute some new perspectives. I shall conclude with a partial list of historical issues that remain to be explored.

11.2 Singular Lagrangian Prehistory

All attempts to invent a Hamiltonian version of singular Lagrangian models are based either explicitly or implicitly on Emmy Noether's remarkable second theorem (Noether 1918). I state the theorem using the notation for variables employed by

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Bergmann in his first treatment of singular systems (Bergmann 1949b). Denote field variables by $y_A (A = 1, \dots, N)$, where N is the number of algebraically independent components, and let x represent the independent variables or coordinates. Noether assumes that n is the highest order of derivatives of y_A appearing in the Lagrangian, $L(y_A, y_{A,\mu}, \dots, y_{A,\mu_1 \dots \mu_n})$, but I will assume that $n = 1$. The extension of the theorem to higher derivatives is straightforward. Then for an arbitrary variation $\delta y_A(x)$, after an integration by parts we have the usual identity

$$L^A \delta y_A \equiv \delta L - \frac{\partial}{\partial x^\mu}\left(\frac{\partial L}{\partial y_{A,\mu}} \delta y_A\right), \tag{11.1}$$

where the Euler–Lagrange equations are

$$L^A := \frac{\partial L}{\partial y_A} - \frac{\partial}{\partial x^\mu}\left(\frac{\partial L}{\partial y_{A,\mu}}\right) = 0. \tag{11.2}$$

Now suppose that the action is invariant under the infinitesimal coordinate transformation $x'^\mu = x^\mu + \xi^\mu(x)$. Invariance is defined by Noether as follows:

$$\int_{\mathcal{R}'} L(y'_A, y'_{A,\mu}) d^4x' = \int_{\mathcal{R}} L(y_A, y_{A,\mu}) d^4x. \tag{11.3}$$

(The notion of invariance was extended later, as we shall see below, to include a possible surface integral.) Crucial to this definition is the fact that the Lagrangian is assumed not to have changed its functional form, guaranteeing that this transformation does not change the form of the equations of motion, i.e., it is a symmetry transformation. Noether writes $\delta y_A(x) := y'_A(x') - y_A(x)$, and therefore $y'_A(x) = y_A(x - \xi) + \delta y_A(x)$. She then defines

$$\overline{\delta} y_A(x) := y'_A(x) - y_A(x) = \delta y_A(x) - y_{A,\mu}(x)\xi^\mu(x). \tag{11.4}$$

This $\overline{\delta}$ notation was appropriated by Bergmann in his 1949 paper and retained throughout his life. It is, of course, the Lie derivative with respect to the vector field $-\xi^\mu$, a terminology introduced in (Sledbodzinski 1931). Returning to the elaboration of Noether's theorem and using this notation, we may rewrite the invariance assumption (11.3) as

$$\overline{\delta} L \equiv -\frac{\partial}{\partial x^\mu}\left(L\xi^\mu\right), \tag{11.5}$$

so that under a symmetry transformation the identity (11.1) becomes

$$L^A \overline{\delta} y_A \equiv \frac{\partial}{\partial x^\mu}\left(-\frac{\partial L}{\partial y_{A,\mu}} \overline{\delta} y_A - L\xi^\mu\right). \tag{11.6}$$

Next, let us assume that $\overline{\delta}$ variations of y_A are of the form

$$\overline{\delta} y_A = {}^0 f_{Ai}(x, y, \dots)\xi^i(x) + {}^1 f^\nu_{Ai}(x, y, \dots)\xi^i_{,\nu}(x), \tag{11.7}$$

where we have admitted the possibility of additional non-coordinate gauge symmetries by letting the index i range beyond 3. We are finally in position to state (and prove) Noether's second theorem: Perform an integration by parts on the left-hand side of (11.6) using (11.7); then for functions ξ^i that vanish on the integration boundary it follows that

$$L^{A\,0} f_{Ai} - \frac{\partial}{\partial x^\nu}\left(L^{A\,1} f^\nu_{Ai}\right) \equiv 0. \tag{11.8}$$

In vacuum general relativity these are the contracted Bianchi identities.The derivation from general coordinate symmetry had already been anticipated by Hilbert in 1915 in a unified field-theoretic context (Hilbert 1915).[1] Weyl applied a similar symmetry argument in 1918 (Weyl 1918). He adapted Noether's theorem to a gravitational Lagrangian $\mathcal{L}_\mathcal{W}$ from which a total divergence has been subtracted so that the highest order of derivatives appearing in it are $g_{\mu\nu,\alpha}$. $\mathcal{L}_\mathcal{W}$ is no longer a scalar density, but the extra divergence term can easily be incorporated in its variation,

$$\mathcal{L}_\mathcal{W} = \sqrt{-g}R - \left(\sqrt{-g}g^{\mu\nu}\Gamma^\rho_{\mu\rho} - \sqrt{-g}g^{\mu\rho}\Gamma^\nu_{\mu\rho}\right)_{,\nu} = \sqrt{-g}g^{\mu\nu}\left(\Gamma^\sigma_{\rho\sigma}\Gamma^\rho_{\mu\nu} - \Gamma^\sigma_{\mu\rho}\Gamma^\rho_{\nu\sigma}\right). \tag{11.9}$$

Bergmann and his collaborators later worked with this Lagrangian. It appears in his 1942 textbook (Bergmann 1942). In 1921 Pauli applied similar symmetry arguments, citing Hilbert and Weyl, but curiously never mentioning Noether (Pauli 1921). Pauli is an important link in this story. In his groundbreaking paper on constrained Hamiltonian dynamics, Leon Rosenfeld writes that it was Pauli who suggested to him a method for constructing a Hamiltonian procedure in the presence of identities (Rosenfeld 1930).

> Bei der näheren Untersuchung dieser Verhältnisse an Hand des besonders lehrreichen Beispieles der Gravitationstheorie, wurde ich nun von Prof. Pauli auf das Prinzip einer neuen Methode freundlichst hingewiesen, die in durchaus einfacher und natürlicher Weise gestattet, das Hamiltonsche Verfahren beim Vorhandensein von Identitäten auszubilden [. . .]

Rosenfeld did indeed make astounding progress in constructing a gravitational Hamiltonian. Full details are reported elsewhere (Salisbury 2009), but its relevance specifically to the work of the Syracuse "school" will be addressed below.

11.3 A Brief Bergmann Biography

Peter Bergmann was born in 1915 in Berlin Charlottenburg. His mother, Emmy (Grunwald) Bergmann, was one of the first female pediatricians in Germany. In 1925 she was also the founder of the second Montessori school in Germany, in Freiburg, where she moved with her son and daughter in 1922. She had taken a course in Amsterdam in the winter of 1923/1924 with Maria Montessori. The chemist Max Bergmann, Peter's father, was a student of and collaborator with the 1902 Nobel prize winner in chemistry, Emil Fischer. In 1923 he was appointed the first Director of the Kaiser Wilhelm Institut für Lederforschung in Dresden. Despite the personal

[1] For a thorough discussion see (Renn and Stachel 2007).

intervention of then-president of the Kaiser Wilhelm Gesellschaft, Max Planck, he was removed from this position by the new Hitler regime in 1933. He then assumed a position in New York City at what was to become Rockefeller University in 1936. Max Bergmann is recognized as one of the founders of modern biochemistry. Peter's aunt, Clara Grunwald, was the founder of the Montessori movement in Germany. He had fond memories of visits with his mother's eldest sister in Berlin.[2] He clearly had benefited from Montessori methods, as attested by his aunt in references to him in her letters written from what had become a forced labor camp near Fürstenwald just outside of Berlin (Grunwald 1985). In 1943 Clara Grunwald perished with her students in Auschwitz.

After completing his doctorate in physics at Charles University in Prague in 1936, Peter Bergmann was appointed an assistant to Albert Einstein at the Institute for Advanced Study in Princeton. He worked with Einstein on unified field theory until 1941. There followed brief appointments at Black Mountain College, Lehigh University, Columbia University, and the Woods Hole Oceanographic Institute. In 1947 he joined Syracuse University, where he remained until his retirement in 1982. From 1963 to 1964 he also held an appointment as Chair of the Department of Physics of the Belfer Graduate School of Science of Yeshiva University. He remained active for many years after his retirement with a research appointment at New York University. Syracuse became the center for relativity research in the United States during the 1950s and early 1960s, bringing virtually all the leading relativists in the world for either brief or prolonged interaction with Bergmann and his collaborators. Bergmann concentrated from the beginning on the conceptual and technical challenges of attempts to quantize general relativity. Not unlike Einstein himself, his deep physical intuition was founded on hands-on laboratory experience, in his case extending back to "enjoyable" laboratory courses in physics and chemistry in 1932 as an undergraduate at the Technical University in Dresden (Bergmann Archive). Later on he expressed appreciation for the opportunity that teaching at the graduate level had given him to explore domains outside of relativity. His two-volume set of lectures on theoretical physics provides magisterial, lucid surveys of the field (Bergmann 1949a, 1951), and it is lamentable that it is now out of print. In fact, the purely mathematical aspect of relativity was not especially appealing to him, and he tended not to work closely with visitors in the 1960s who approached the subject from this perspective.[3] For additional biographical material see my short sketch (Salisbury 2005) and a longer discussion by Halpern (2005).

11.4 1949–1951

Bergmann's aim in his 1949 paper is to establish a general, classical, canonical framework for dealing with a fairly narrow set of generally covariant dynamical systems, but a set that includes as a special case general relativity described by the

[2] Personal communication.
[3] Engelbert Schucking, personal communication.

Lagrangian $\mathcal{L}_{\mathcal{W}}$ above. He assumes that, under the infinitesmal general coordinate transformation $x'^{\mu} = x^{\mu} + \xi^{\mu}(x)$, the $\overline{\delta}$ transformations are given by

$$\overline{\delta} y_A = F_{A\mu}{}^{B\nu} y_B \xi^{\mu}_{,\nu} - y_{A,\mu}\xi^{\mu}, \tag{11.10}$$

where the $F_{A\mu}{}^{B\nu}$ are constants. Noether is not cited in this paper, surely because at this time her theorem was common knowledge.[4] A principal concern from the start is with the group structure of these symmetry transformations, and with the requirement that canonically-realized variations faithfully reproduce the $\overline{\delta}$ variations.

Due to the intended use of the Lagrangian $\mathcal{L}_{\mathcal{W}}$, an additional term will appear on the right-hand side of the invariance assumption (11.5). This eventuality is accommodated by Bergmann with the assumption that $\overline{\delta} L \equiv Q^{\mu}_{,\mu}$. Rather than consider ξ^{μ} that vanish on the integration boundaries, he equivalently requires the identical vanishing of that contribution to the duly-rewritten (11.6), which now cannot be written as a total divergence. Thus he obtains the generalized contracted Bianchi identity (11.8), which for the variations (11.10) takes the form

$$\left(F_{A\mu}{}^{B\nu} y_B L^A\right)_{,\nu} + y_{A,\mu} L^A \equiv 0. \tag{11.11}$$

It is at this stage that new information is mined from the invariance of the Lagrangian. In particular, Bergmann noted that the third time derivative of y_A appears linearly in (11.11), and its coefficient must vanish identically,

$$F_{A\mu}{}^{B0} \Lambda^{AC} y_B \equiv 0, \tag{11.12}$$

where

$$\Lambda^{AB} := -\frac{\partial^2 L}{\partial \dot{y}_A \partial \dot{y}_B} \tag{11.13}$$

is minus the Legendre matrix.[5] Thus the Legendre matrix possesses null vectors. This is the signature of singular Lagrangians.

Bergmann deduces several interrelated consequences. Firstly, since by assumption the Euler–Lagrange equations are linear in $\ddot{y}_A$, with the linear term of the form $\Lambda^{AC}\ddot{y}_C$, the following four linear combinations of the equations of motion do not contain accelerations:

$$y_B F_{A\mu}{}^{B0} L^A = 0. \tag{11.14}$$

Therefore, the evolution from an initial time will not be uniquely fixed through an initial choice of y_A and $\dot{y}_A$. Secondly, it will not be possible to solve for velocities in terms of canonical momenta $\pi^A := \partial L/\partial \dot{y}_A$ since the matrix Λ^{AB} cannot be inverted. Thirdly, since

$$y_B F_{A\mu}{}^{B0} \frac{\partial \pi^A}{\partial \dot{y}_C} \equiv \frac{\partial}{\partial \dot{y}_C}\left(y_B F_{A\mu}{}^{B0} \pi^A\right) \equiv 0, \tag{11.15}$$

4 As far as I can tell, Bergmann's first explicit published reference to Noether's theorem occurs in his *Handbuch der Physik* article on general relativity (Bergmann 1962).

5 A "dot" signifies a derivative with respect to time.

straightforward integration yields a constraining relation among the momentum π^A and configuration variables y_B.

Bergmann was not aware that Léon Rosenfeld had obtained the same results in 1930, but working directly with the identity (5). (Rosenfeld 1930) Rosenfeld used the fact that in this identity the coefficients of each order of time derivative of the arbitrary functions ξ^i must vanish. He did not make explicit use of the Bianchi identities.

Although the stated central objective of Bergmann's paper was to prepare the ground for a full-scale quantization of the gravitational field, he did note that the canonical phase space approach offered a potentially new method for solving the classical particle equation of motion problem. Indeed, he expressed a hope shared by Einstein that, by avoiding singular field sources at the locations of point particles, it might be possible to eliminate singularities in an eventual quantum-gravitational field theory. This hope led in the second paper, henceforth denoted BB49, co-authored with Brunings, to the introduction of a parameterized formalism, in which spacetime coordinates x^μ themselves became dynamical variables (Bergmann and Brunings 1949). For the further development of the constrained dynamical formalism this was an unnecessary computational complication, yet several important results were obtained. In the parameter formalism, the Lagrangian is homogeneous of degree one in the velocities. Consequently, the Hamiltonian density $\mathcal{H}$ vanishes identically. It was possible to find immediately seven functions of the y_a and conjugate momenta π^b whose vanishing follows from the Legendre map $\pi^a(y, \dot{y}) := \partial L/\partial \dot{y}_a$. (The range of the index a has been expanded by four to accommodate the spacetime coordinates.) BB49 recognized that the pullback of the Hamiltonian under the Legendre map yielded a null vector of the Legendre matrix,

$$0 \equiv \frac{\partial}{\partial \dot{y}_a}\mathcal{H}\left(y, \pi(y, \dot{y})\right) = \frac{\partial \mathcal{H}}{\partial y_b}\Lambda^{ba}. \tag{11.16}$$

But the homogeneity of the Lagrangian implies that the velocities are also components of a null vector. It follows that one may set $\dot{y}_a = \partial\mathcal{H}/\partial\pi^a$. Dirac would soon reach the same conclusion in his parameterized flat spacetime models (Dirac 1950). Apparently unbeknownst to either of the parties, Rosenfeld had already shown in 1930 that a relation of this form more generally reflects the freedom to alter the velocities without affecting the momenta, albeit in models with Lagrangians quadratic in the velocities (Rosenfeld 1930). Next, considering variations of $\mathcal{H}$ at a fixed time, and using the Euler–Lagrange equations, Bergmann and Brunings obtained the "usual" additional Hamiltonian dynamical equations $\dot{\pi}^a = -\frac{\partial\mathcal{H}}{\partial y_a}$. BB49 do note that there is considerable freedom in the choice of the vanishing Hamiltonian: Given any $\mathcal{H}$ resulting from the homogeneity of the Lagrangian, one may multiply by an arbitrary function of the spacetime coordinates and add arbitrary linear combinations of the remaining seven constraints without altering the canonical form of the Hamiltonian equations. However, they do appear to claim erroneously that the vanishing of all of these possibilities is preserved under the evolution of a fixed Hamiltonian. Unfortunately, this renders untenable the proposed Heisenberg picture quantization, in which the quantum states are annihilated by all of the constraints $\mathcal{L}_{\mathcal{W}}$.

In this paper, we find the first statement of the requirement of projectability under the Legendre transformation from configuration-velocity space to phase space. Only those functions Ψ that are constant along the null directions u_a of Λ^{ab} have a unique counterpart in phase space since

$$u_a \frac{\partial}{\partial \dot{y}_a} \Psi(y, \pi(y, \dot{y})) = u_a \frac{\partial \Psi}{\partial p^b} \Lambda^{ba} = 0. \tag{11.17}$$

This requirement remained a concern until it appeared to have been resolved in 1958 through the elimination of lapse and shift variables, as described below. Only much later was its relevance to the canonical symmetry group understood (Lee and Wald 1990; Pons et al. 1997).

The explicit expression for the Hamiltonian was obtained by Bergmann, Penfield, Schiller, and Zatzkis in the next paper, henceforth denoted BPSZ50 (Bergmann et al. 1950). Because of the ensuing complications in the parameterized formalism, the solution was a daunting task. The work focuses on an algorithm for transforming the Legendre matrix into a "bordered" form in which the final eight rows and columns are zero. We will not address the details here since much of the technology was rendered superfluous by the discovery by Penfield, one of Bergmann's students, that the parameterization could be profitably dispensed with. Josh Goldberg vividly recalls the ensuing excitement; it was he who communicated the news to their approving mentor.[6] Penfield worked with a quadratic Lagrangian of the form

$$L = \Lambda^{A\rho B\sigma}(y) y_{A,\rho} y_{B,\sigma}, \tag{11.18}$$

so

$$\pi^A = 2\Lambda^{A0Ba} y_{B,a} + \Lambda^{AB} \dot{y}_B, \tag{11.19}$$

where $\Lambda^{AB} := 2\Lambda^{A0B0}$ is the Legendre matrix (Penfield 1951). His task was to find the appropriate linear combination of the $\dot{y}_A$ such that Λ^{AB} is transformed into a bordered matrix. In somewhat more technical terms, he sought a linear transformation in the tangent space of the configuration-velocity space such that each null vector acquires a single non-vanishing component. This procedure had already been undertaken in BPSZ50, but its implementation in this context was much simpler.

Indeed, it is immediately clear from (11.19) that, once a particular solution for $\mathcal{H}$ is found, resulting in a fixed $\dot{y}_A$, any linear combination of the remaining constraints may be added to H since, as also noted in BPSZ50, the additional terms do not change π^B. (Recall that the gradients of constraints with respect to momenta are null vectors.)

As pointed out already in BB49, additional gauge symmetry can easily be incorporated into the formalism, resulting in as many new constraints as there are new gauge functions. Thus both BB49 and BPSZ50 produced Hamiltonians for gravity coupled to electromagetism.

[6] J. Goldberg, personal communication.

At some time in 1950, the Syracuse group became aware of the pioneering work of Leon Rosenfeld. Reference to Rosenfeld appears in a 1950 *Proceedings* abstract (Bergmann 1950). James Anderson thinks it is possible that he brought the work to Bergmann's attention,[7] and Bergmann showed the paper to Ralph Schiller. In fact, according to Schiller the paper inspired his doctoral thesis.[8] In any case, the culminating paper of this period by Bergmann and Anderson, henceforth denoted BA51 (Bergmann and Anderson 1951), was written after this discovery, and it does appear that the authors were motivated by it to broaden the scope of their published investigations of constrained Hamiltonian dynamics. In particular, in addition to abandoning the parameterized theory, BA51 contemplated more general symmetry transformations, similar to those of Rosenfeld:

$$\overline{\delta} y_A = {}^0f_{Ai}(x, y, \ldots)\xi^i(x) + \cdots + {}^Pf_{Ai}^{\nu_1 \cdots \nu_P}(x, y, \ldots)\xi^i_{,\nu_1 \cdots \nu_P}(x). \tag{11.20}$$

The BA51 collaboration was a watershed, in which most of the basic elements of the formalism were presented. For the first time in this paper the question was asked whether coordinate-transformation-induced variations of the momenta are realizable as canonical transformations. BA51 assumed that the canonical generator density $\mathcal{C}$ of these symmetry transformations could be written as

$$\mathcal{C} = {}^0A_i\xi^i + {}^1A_i\frac{\partial \xi^i}{\partial t} + \cdots + {}^PA_i\frac{\partial^P \xi^i}{\partial t^P}, \tag{11.21}$$

where the ${}^MA_i$ are phase space functions and t is the coordinate time. Thus it was necessary to show, as they did, that the momenta variations do not depend on time derivatives of ξ of order higher than P; the potential offending term in $\overline{\delta}\pi^A$ is $2\Lambda^{AB}(-1)^P\, {}^Pf_{Bi}^{\nu_1 \cdots \nu_P}\frac{\partial^P \xi^i}{\partial t^P}$, but $\Lambda^{AB}\, {}^Pf_{Bi}^{\nu_1 \cdots \nu_P}$ vanishes identically, extending the null vector relation (11.12) to this model. Most importantly, BA51 argued that, since the commutator of transformations generated by $\mathcal{C}$'s must be of the same form, the ${}^MA_i$'s must form a closed Poisson bracket algebra. Furthermore, they were able to show that the ${}^PA_i$ are the constraints that follow from the momenta definitions. For these they introduced the term "primary constraints." They showed that, in order for these constraints to be preserved under time evolution, all of the ${}^MA_i$'s must be required to vanish; again, according to their terminology, ${}^{P-1}A_i$ is a secondary constraint, ${}^{P-2}A_i$ tertiary, etc. The argument employed here is similar to one used by Rosenfeld. Up to this point Rosenfeld's results are similar. He does not, however, take the next step, in which BA51 derives a partial set of Poisson relations among the ${}^MA_i$'s. All of these results are displayed explicitly for gravity and a generic generally-covariant model that includes Einstein's gravity as a special case.

11.5 Preview of Some Later Developments

It is not possible to do justice to Bergmann's complete oeuvre in constrained Hamiltonian dynamics in this paper. I will just briefly mention two important developments

[7] Personal communication.
[8] Personal communication.

that are treated in detail elsewhere, and then conclude with a teaser of contemporary importance. Much effort was expended in Syracuse in the 1950s in constructing gravitational observables: functions of the canonical variables that are invariant under the full group of general coordinate transformations. In 1958 Paul Dirac published his simplified gravitational Hamiltonian, achieved through a subtraction from the Lagrangian that resulted in the vanishing of four momenta (Dirac 1958).[9] He argued that the corresponding configuration variables, the lapse and shift functions, could then simply be eliminated as canonical variables. There remained a puzzle over the precise nature of the canonical general coordinate symmetry group. Bergmann and Komar made considerable headway in describing this group in 1972 (Bergmann and Komar 1972). They showed in particular that the group must be understood as a transformation group on the metric field. This view was forced on them by the observation that the group involved a compulsory dependence on the metric, and it was manifested in part by the appearance of metric components in the group Poisson bracket algebra. A close relation exists between these developments and the "problem of time" in general relativity. Are invariants under the action of the group necessarily independent of time? The issue is addressed in an early exchange between Bergmann and Dirac, with which I will close (Bergmann Archive). In a letter to Dirac dated October 9, 1959, Bergmann wrote: "When I discussed your paper at a Stevens conference yesterday, two more questions arose, which I should like to submit to you: To me it appeared that because you use the Hamiltonian constraint H_L to eliminate one of the non-substantive field variables K, in the final formulation of the theory your Hamiltonian vanishes strongly, and hence all the final variables, i.e. $\tilde{e}^{rs}, \tilde{p}^{rs}$, are 'frozen,' (constants of the motion). I should not consider that as a source of embarrassment, but Jim Anderson says that in talking to you he found that you now look at the situation a bit differently. Could you enlighten me?" Here is Dirac's response, dated November 11, 1959: "If the conditions that you introduce to fix the surface are such that only one surface satisfies the condition, then the surface cannot move at all, the Hamiltonian will vanish strongly and the dynamical variables will be frozen. However, one may introduce conditions which allow an infinity of roughly parallel surfaces. The surface can then move with one degree of freedom and there must be one non-vanishing Hamiltonian that generates this motion. I believe my condition $g_{rs}p^{rs}$ is of this second type, or maybe it allows also a more general motion of the surface corresponding roughly to Lorentz transformations. The non-vanishing Hamiltonian one would get by subtracting a divergence from the density of the Hamiltonian."

[9] At about the same time, the same Hamiltonian was obtained independently by B. DeWitt and also by J. Anderson. Beginning with their first paper in 1959, and culminating in 1962, Arnowitt, Deser, and Misner produced an equivalent geometrically based gravitational Hamiltonian formalism (Arnowitt et al. 1962).

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